4-2 Optical Heterodyne Detection for 60-GHz-Band Radio-on-Fiber Systems

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A novel method for optical heterodyne detection in millimeter-wave-band radio-on-fiber systems is described. The key to detection is to use a remote dual-mode local light. Although the light is free-running, our method is in principle free from laser phase noise. This method is also theoretically immune from the fiber-dispersion effect, because only two components of the optical signal are selected out by the local light to demodulate themselves. This immunity persists even if the transmitted optical signal is in the double-sideband format. We derive the theoretical limit of the system performance and experimentally demonstrate a 25-km-long fiber-optic transmission and the optical heterodyne detection of a 59.6-GHz radio-on-fiber signal using 155.52-Mb/s DPSK-formatted data.

Keywords
Optical heterodyne detection, Dual-mode local light, Radio-on-fiber, Optical fiber dispersion, Laser phase noise

1 Introduction

External modulation will provide the best solution to configure simpler base stations (BSs) in future microwave or millimeter-wave (mm-wave) radio-on-fiber systems, leading to cost-effective system configuration. We have reported on radio-on-fiber systems that use a 60-GHz-band electroabsorption modulator (EAM) for downlinking and uplinking. To receive optical power large enough to get high transmission quality, using some optical amplifiers is necessary in the optical link. However, accumulated amplified spontaneous-emission (ASE) noise from the optical amplifiers cannot be removed completely, in analog optical communication systems, such as radio-on-fiber applications. The noise fatally affects the photodetected signal, resulting in a degradation in system performance. The theory that optical coherent detection using a remote local light source has higher-sensitivity to a received optical signal than direct detection under a shot-noise-limited condition is well known. Thus, we expect that if we use the coherent detection technique, we can avoid using optical amplifiers. Laser phase noise and polarization mismatch cause this degradation if we consider using a remote local light source for optical heterodyne detection. Because the polarization mismatch is a common problem for optical fiber systems, we concentrate on using optical heterodyne detection that is insensitive to laser phase noise.

In this paper, we describe a novel method for optical heterodyne detection in radio-on-fiber systems at a 60-GHz band. A free-running dual-mode local light is used to detect a 60-GHz-band radio-on-fiber signal. We show that the system, in principle, is not affected by the phase noises not only for the transmitting laser but also for the remote local one. This scheme is also theoretically immune from the fiber-dispersion effect which causes signal fading, because only two components of the optical signal are selected out by the local light to demodulate themselves. This immunity persists even if the transmitted optical signal is in a double-sideband (DSB) format.
Although some dispersion-free systems have been proposed, they have never essentially solved the ASE problem. We present a principle together with a mathematical description of signals in our method and derive the theoretical limit of the system performance. We experimentally verify the principle of our method by demonstrating a 25-km-long fiber-optic transmission and the optical heterodyne detection of a 59.6-GHz radio-on-fiber signal using 155.52-Mb/s differential-phase-shift-keying (DPSK).

2 Principle

2.1 Mathematical Description

Fig.1 shows the fundamental block diagram of the optical heterodyne detection using dual-mode local light source. The transmitter consists of a single-mode light source and optical external and electrical modulators. Also, the optical heterodyne receiver consists of a dual-mode local light source, optical power combiner, photodetector (PD), phase noise canceling (PNC) circuit, and electrical demodulator.

The complex electric field of the optical carrier, $e_{c1}(t)$, which is oscillated from the single-mode light source in the transmitter, is written as

$$
e_{c1}(t) = E_{c1} \cdot \exp \{ j\varphi_{c1}(t) \}, \tag{1}$$

where $E_{c1}$, $f_{c1}$, and $\varphi_{c1}(t)$ are the amplitude, the center frequency, and the phase noise of the optical carrier, respectively. The radio-frequency (RF) signal, $e_{RF}(t)$, is written as

$$
e_{RF}(t) = V_{RF} \cdot \exp \{ j\varphi_{RF}(t) \}, \tag{3}$$

where $V_{RF}$, $f_{RF}$, and $\varphi_{RF}(t)$ represent the amplitude, the carrier frequency, and the phase-modulated data of the RF signal, respectively. The optical carrier, $e_{c1}(t)$, is modulated by the RF signal, $e_{RF}(t)$, and transmitted over the optical fiber. Taking into account the fiber-dispersion effect, after the propagation in the optical fiber of the length, $L$, the complex electric field of the modulated optical signal is generally written as

$$
e_s(t, L) = \text{MOD} \{ e_{RF}(t) \cdot e_{c1}(t) \cdot e^{-j\beta(f)L} \}
\approx E_{c1} \cdot \sum_{n=-\infty}^{\infty} a_n \cdot e^{j\varphi_{n}(t,L)}, \tag{5}$$

$$
\varphi_{s,n}(t, L) = \varphi_{c1}(t) + n\varphi_{RF}(t) - \beta(f) \cdot L 
\approx 2\pi (f_{c1} + n f_{RF}) \cdot (t - \beta_1 L) 
+ n\theta (t - \beta_1 L) + \psi_{c1} (t - \beta_1 L) 
- \frac{1}{2} \beta_2 (2\pi f_{RF})^2 \cdot L 
+ (2\pi f_{c1} \cdot \beta_1 - \beta_0) \cdot L. \tag{6}$$

Fig.1 Block diagram of proposed optical heterodyne detection scheme
where $L$ corresponds to the group delay time and $\beta_1$ is related with the dispersion, $D$, as

$$
\beta_1 = -\frac{\lambda_2^2}{2\pi c} \cdot D.
$$

$\lambda$ and $c$ are the wavelength in the fiber and the velocity in the vacuum, respectively. Note that the last term in Eq.(6) is constant and independent of time and the last second term represents the chromatic fiber-dispersion effect. Then, MOD[$\cdot$] represents the response function of the optical modulator, depending on the scheme used in the modulator, such as intensity, phase, or frequency. Therefore, $a_n$ is given as the Fourier coefficient of MOD[$\cdot$]. If the intensity modulation is performed and the modulation index, $m_{IF}$, is small, for example, then the Fourier coefficients are written approximately as

$$
a_0 \approx \sqrt{\frac{1}{2}} \left[ 1 - \left( \frac{m_{IM}}{4} \right)^2 \right],
$$

$$
a_{\pm n} \approx (-1)^{n-1} \sqrt{\frac{1}{2}} \left( \frac{2(n - 3)!}{n!} \right) \left( \frac{m_{IM}}{4} \right)^n \quad \text{for } n \neq 0,
$$

where $(2n + 1)$ $\equiv$ $1 \cdot 3$ $\equiv$ $(2n + 1)$, $(2n) \equiv$ $2 \cdot 4$ $\equiv$ $(2n)$, and $0$ $\equiv$ $(-1)$ $\equiv$ $1$. The details are described in Appendix A. If the input amplitude of the modulating signal is small ($V_{RF} \ll 1$), then $e_s(t)$ is approximated as

$$
e_s(t,L) \approx E_3 \cdot a_{-1} \cdot e^{j\varphi_{s-1}(t,L)} + E_3 \cdot a_0 \cdot e^{j\varphi_{s0}(t,L)} + E_3 \cdot a_1 \cdot e^{j\varphi_{s1}(t,L)}.
$$

The free-running dual-mode local light that has a frequency separation of $f_{LO}$ is used to detect the radio-on-fiber signal, $e_s(t,L)$, in the optical heterodyne receiver. Here, the dual-mode local light source is considered to have a frequency separation that is either highly stabilized or jitter-free. The complex electric field of the dual-mode local light, $e_l(t)$, is written as

$$
e_l(t) = \frac{1}{\sqrt{2}} E_2 \cdot \exp \{ j\varphi_{-1}(t) \} + \sqrt{2} E_2 \cdot \exp \{ j\varphi_{+1}(t) \},
$$

$$
\varphi_{-1}(t) = 2\pi \left( f_a - f_{LO} \right) t + \phi_{a2}(t),
$$

$$
\varphi_{+1}(t) = 2\pi \left( f_a + f_{LO} \right) t + \phi_{a2}(t),
$$

where $E_2$, $f_2$, and $\phi_{a2}(t)$ are the amplitude, the center frequency, and the phase noise of the dual-mode local light, respectively. The dual-mode local light, $e_l(t)$, is combined with the received optical signal, $e_s(t,L)$, as shown in Fig.2. The optical heterodyne detection is performed using a square-law response of the PD. Let us assume that the polarizations between the received optical signal and the local light are matched. Then, the photocurrent becomes

$$
e_p(t,L) = \mathcal{R} \sqrt{2} P_1 P_2 a_0 e^{j(\varphi_{s0}(t,L) - \varphi_{l0}(t))}
$$

$$+ \mathcal{R} \sqrt{2} P_1 P_2 a_{-1} e^{j(\varphi_{s-1}(t,L) - \varphi_{l-1}(t))}
$$

$$+ \mathcal{R} \sqrt{2} P_1 P_2 a_1 e^{j(\varphi_{s1}(t,L) - \varphi_{l+1}(t))}
$$

$$+ \mathcal{R} \sqrt{2} P_1 P_2 a_{-1} e^{j(\varphi_{s-1}(t,L) - \varphi_{l-1}(t))}
$$

$$+ \mathcal{R} \sqrt{2} P_1 P_2 a_1 e^{j(\varphi_{s1}(t,L) - \varphi_{l+1}(t))}
$$

In the above equation, $P_1$ and $P_2$ are the photodetected power of the signal and local light, respectively and are given as

$$
P_1 = \int_A \frac{|e_s(t)|^2}{2Z_w} dA,
$$

$$
P_2 = \int_A \frac{|e_l(t)|^2}{2Z_w} dA.
$$

Here, $A$ and $Z_w$ represent the photodetecting effective area and the wave impedance, respectively. Note that $\mathcal{R}$ is the responsivity of the PD. We focus on the two phase compo-
\[
\varphi_{x0}(t, L) = \varphi_0(t)
\]
\[
= 2\pi \left( \Delta f + \frac{f_{LO}}{2} \right) (t - \beta_1 L) + \Delta \phi_x(t, L) + \frac{2\pi}{2} \left( \Delta f + \frac{f_{LO}}{2} \right) \beta_1 - \beta_0 L.
\]

\[
\varphi_{x1}(t, L) - \varphi_{x1}(t)
\]
\[
= 2\pi \left( \Delta f + f_{RF} - \frac{f_{LO}}{2} \right) (t - \beta_1 L) + \theta(t - \beta_1 L) + \Delta \phi_x(t, L)
\]
\[
= \left\{ \frac{1}{2} \beta_2 (2\pi f_{RF})^2 \right\} \cdot L
\]
\[
+ \left\{ \frac{2\pi}{2} \left( \Delta f + f_{RF} - \frac{f_{LO}}{2} \right) \beta_1 - \beta_0 \right\} \cdot L
\]

where \( f \) and \( \omega \) \((t, L)\) are defined as
\[
\Delta f = f_{LO} - f_{RF},
\]
\[
\Delta \phi_x(t, L) = \phi_x(t - \beta_1 L) - \varphi_{x0}(t).
\]
this method has the same effect as in the optical SSB-formatted transmission. Third, the filtering is not performed in the optical domain but in the electrical domain to select the desired signal components. Our method will be able to provide fine and tunable filtering for the desired optical components by making good use of both the local light and the electrical BPF. This is because the frequency stability and controllability of the lasers have recently progressed in conjunction with the development of dense wavelength division multiplexing (DWDM) technologies.

2.2 Theoretical System Performance

We will now describe the SNR of the down-converted signal, \( i_2(t) \), in the system using our detection method. As shown in the previous section, there was no fiber-dispersion effect in the demodulation, and therefore, we will omit the propagation constant, \( \beta(f) \).

Let us rewrite the photodetected signal with no data (\( B(t) \square 0 \)) and the local components in the real part

\[
s(t) = a_s \cdot \cos \left( 2\pi \left( \Delta f + f_{RF} - \frac{f_{LO}}{2} \right) t \right) + \Delta \phi(t),
\]

where \( \Delta \phi(t) \) is the phase fluctuation due to the frequency instability of the laser. The photodetected signal appears at \( f_{RF} \). The photocurrent is expressed as

\[
i_3(t) = \frac{1}{2} R \sqrt{2 P_{ol} P_{a1}} \cdot a_1 t.
\]

When the shot noise is dominant, the noise spectral density, \( \square \), is given as

\[
\eta = \frac{1}{4} e R P_{ol} \left( 1 + \frac{1}{2} P_{a1} \right).
\]

If the input put into the multiplier is optimally bandwidth-limited, then the SNR becomes (see Appendix C)

\[
\frac{S}{N} = \frac{1}{2} \alpha P_{ol} \cdot \frac{a_0^2 a_1^2}{B_{IF} \cdot a_0^2 + a_1^2} \cdot \frac{1}{2 n_{sp} \left( 1 - \frac{1}{G} \right)}.
\]

Here, \( \square = \frac{R}{e} \) is the sensitivity of the PD. Under the local shot-noise-limited environ-

\[
\frac{S}{N} = \frac{1}{2} \alpha P_{ol} \cdot \frac{a_0^2 a_1^2}{B_{IF} \cdot a_0^2 + a_1^2} \cdot \frac{1}{2 n_{sp} \left( 1 - \frac{1}{G} \right)}.
\]

If the gain in the optical amplifier is large enough to suppress the other noise components (\( G \square 1 \)), then the theoretical limit of SNR is given as

\[
\frac{S}{N} = \frac{1}{2 n_{sp} \cdot B_{IF} \cdot a_0^2 + a_1^2}.
\]

Comparing Eqs. (31) and (36), we can see that the SNR for our method is improved by the amount of \( n_{sp} \). Note that \( n_{sp} \) represents the ASE noise, which is accumulated in the link and fatally affects system performance.

\[
\frac{S}{N} = \frac{1}{2 n_{sp} \cdot B_{IF} \cdot a_0^2 + a_1^2}.
\]
3 Experiment

3.1 Experimental Setup

To verify the principle of our method, we conducted an experiment using the following setup. As shown in Fig.5, the optical system in the setup consisted of a distributed feedback laser diode (LD1), 60-GHz-band electroabsorption modulator (EAM), 3-dB optical coupler, tunable laser diode (LD2), LiNO3 modulator (EOM), erbium-doped fiber amplifier (EDFA), optical isolator, PD, and two polarization controllers (PCs). The PNC based on the electrical square-law detector consists of two electrical amplifiers and an electric mixer (the bandwidth of the RF, LO, and IF ports were 5–18 GHz, 5–18 GHz, and DC–3 GHz, respectively).

An optical carrier \( f_{c1} \) was intensity modulated using a 59.6-GHz signal \( f_{RF} \) by the EAM. The mm-wave signal was DPSK-encoded at the data rate of 155.52 Mb/s (PRBS = 2⁸ - 1). An optical local tone \( f_{c2} \) and the optical signal were combined via the 3-dB coupler and optically heterodyne-detected by the PD. The tone was modulated using a 28.5-GHz \( f_{LO}/2 \) sinusoidal wave at the DC bias of \( V_0 \), which enables the suppression of the optical carrier. The EDFA was used only to amplify the optical local tone. The optically heterodyne-detected signal was amplified in the PNC by the first electric amplifier with the bandwidth of DC–26.5 GHz, was power-divided, put in the electric mixer, and amplified again by the following amplifier (2–4 GHz). The 2.6-GHz \( f_{IF} \) IF signal obtained after mixing was demodulated by a DPSK demodulator to extract the data and the clock.

3.2 Experimental Results

The measured optical spectrum is shown in Fig.6. The thick and thin lines represent the optical signal and the dual-mode local tone in front of the PD, respectively. The wavelengths of LD1 and LD2 were 1550.27 nm \( c/f_{c1} \) and 1550.17 nm \( c/f_{c2} \), respectively. The optical-insertion losses of the EAM at a bias of \( V_1 \) and the EOM at a bias of 1.5 V were 11 and 25 dB, respectively.
power of $f_{RF}$ applied to the EAM was 6 dBm at the same bias, corresponding to a modulation depth of about 44%. In the optical local tone, the carrier still remained but could be removed using the present allocation of $f_{c1}$ and $f_{c2}$, as shown in Fig.3.

The photodetected signals before and after mixing were also measured. In Fig.7, the photodetected signals appear at around 12.4 GHz [= $f_i + f_{LO}/2$], 15.0 GHz [= $f_i + f_{RF} - f_{LO}/2$], and 16.2 GHz [= $- f_i$]. Note that $f_i < f_{c2}$. Large phase noises caused by the individually driven LD1 and LD2 were observed from the measured linewidths because the interval frequency between LD1 and LD2 were not controlled. As shown in Fig.7(b), however, a 2.6-GHz IF signal was stably generated. The observed linewidth was less than 30 Hz, and the measured SSB phase noise was less than -73 dBc/Hz at 10 kHz apart from the carrier, despite the linewidths of LD1 and LD2 being 5 MHz and 100 kHz, respectively. Hence, we successfully demonstrated the phase-noise cancellation.

We also measured the BERs as a function of the optical signal power put into the PD. Fig.8 (a) shows the BERs after the 25-km-long SMF transmission and the BERs for the back-to-back, which are plotted as a function of the optical signal power received by the PD. Optical local power put into the PD was -10.0 dBm. RF input power put into EAM of $f_{RF}$ was 6.3 dBm. No BER floor was observed. The minimum optical signal power that was received to get the BER of $10^{-9}$ was -16.0 dBm. Compared with the back-to-back case, the small power penalty was presumably due to the polarization matching error. When the RF power was below 5.5 dBm, the BER floor appeared. This was due to the ASE noise from the EDFA that boosted the optical local tone. However, the appearance of the BER floor can never pose a serious problem because it can be circumvented by using either a 2-mode DBR-MLLD or a 2-mode injection-locked FP-LD as the dual-mode local light source. Fig.8 (b) shows the BERs as a function of the RF power into the EAM after the 25-km-long SMF transmission and for the back-to-back. The optical received power put into the PD was fixed to be -16.0 dBm. The BER floor did not appear. The
minimum RF input power to get the BER of $10^{-9}$ after the 25-km-long SMF transmission was also 6.3 dBm. Compared with the back-to-back case, no serious power penalty was observed.

4 Conclusion

A novel method for optical heterodyne detection in 60-GHz-band radio-on-fiber systems has been described. The key to detection was to photodetect the radio-on-fiber signal by using a dual-mode local light source at the receiver side. The principle and mathematical description for the detection method has been theoretically described. The free-running dual-mode local light was used to detect a radio-on-fiber signal at a 60-GHz band. When the photodetected signal and reference components multiplied each other, the same amount of the laser phase noise in the components were differentially subtracted. Therefore, our method was in principle free from the laser phase noise. This scheme was theoretically immune from the fiber-dispersion effect, which causes signal fading because only two components of the optical signal are selected out by the local light to demodulate themselves. This immunity persisted even if the transmitted optical signal was in the DSB format. The theoretical limit of the system performance has also been derived. Compared with the conventional IM/DD system using optical amplifiers, we clarified that the proposed detection scheme is superior to the conventional one. To confirm the principle of the proposed scheme, the 25-km-long fiber-optic transmission and the optical heterodyne detection of a 59.6-GHz, 155.52-Mb/s DPSK radio-on-fiber signal has been successfully demonstrated. No serious laser phase noise and fiber-dispersion effects were occurred.

References

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appendix

A   Intensity Modulator

If the intensity modulation is performed, then \( \text{MOD}[e_{RF}(t)] \) is expressed as

\[
\text{MOD}[e_{RF}(t)] = \left[ \frac{1}{2} \{ 1 + m_{IM} \cdot \cos \varphi_{RF}(t) \} \right]^\gamma
\]

where \( m_{IF} \) is the intensity-modulation index as a function of \( V_{RF} \), and \( \varphi \) represents the intensity modulation with the chirp effect, which is defined as

\[
\gamma = \frac{1 + j \alpha_c}{2}.
\]  (A2)

\( \alpha_c \) is the chirp parameter of the optical modulator. We carried out the Fourier expansion of \( \text{MOD}[e_{RF}(t)] \) by using the Binomial theorem. As a result, we got \( a_n \) expressed as

\[
a_{\pm n} = \sum_{n=-\infty}^{\infty} a_n e^{j n \phi_{RF}(t)},
\]

where \( \phi \) is the Gamma function, which has the following relationship.

\[
\Gamma(z + 1) = z \cdot \Gamma(z)
\]  (A4)

If the chirp is negligible (\( \alpha = 0 \)), then \( a_n \) is simplified to

\[
a_{\pm n} = \frac{1}{2^\gamma} \sum_{n=0}^{\infty} \Gamma(\gamma + 1) \frac{(m_{IF}^2)^{2s+n}}{s!(\gamma + 2s - n + 1)}
\]

\[
\times (m_{IF}^2)^{2s+n},
\]  (A3)

where \( \Gamma(\cdot) \) is the Gamma function, which has the following relationship.

\[
\Gamma(z + 1) = z \cdot \Gamma(z)
\]  (A4)

If the chirp is negligible (\( \alpha = 0 \)), then \( a_n \) is simplified to

\[
a_{\pm n} = \frac{1}{2} \sum_{l=0}^{\infty} \frac{(m_{IF}^2)^{2l+n}}{l![2l+n]!!}
\]

\[
\left( (2l + n) - 3 \right)!!
\]

\[
\times \left( \frac{2}{4^{2l+n} \cdot l!(l+n)!} \right),
\]  (A5)

where \( (2n + 1) \) \( \equiv 1 \cdot 3 \) \( (2n + 1) \), \( (2n) \) \( \equiv 2 \cdot 4 \) \( (2n) \), \( 0 \) \( \equiv ( -1) \) \( (2n - 1) \), and \( ( - 2n - 1) \) \( \equiv ( -1)^{n(2n - 1)} \) \( ( -1)^{n} \). Here, we used the following formula for the natural number of \( n \).

\[
\Gamma \left( n + \frac{1}{2} \right) = \frac{(2n - 1)! \sqrt{\pi}}{2^n}
\]  (A6)

If \( m_{IF} \) is small, then the Fourier coefficients are approximately given by

\[
a_0 \approx \sqrt{\frac{1}{2} \left[ 1 - \left( \frac{m_{IF}}{4} \right)^2 \right]},
\]  (A7)

\[
a_{\pm n} \approx (-1)^{n-1} \sqrt{\frac{1}{2} \frac{(2n - 3)!!}{n!} \left( \frac{m_{IF}}{4} \right)^n}
\]

for \( n \neq 0 \).  (A8)

To verify the above calculation, we also compared \( \Box \Box \), for \( m_{IF} = 1 \) with the following analytical solution.

\[
\sqrt{1 + \cos \phi_{RF}(t)} = \sum_{n=-\infty}^{\infty} \hat{a}_n \cdot e^{j n \phi_{RF}(t)}
\]  (A9)

\[
\hat{a}_{\pm n} = \frac{2\sqrt{2}(-1)^{n-1}}{\pi 4^{n^2 - 1}}.
\]  (A10)

As a result, \( a_n \) numerically agreed well with the analytical solution, \( \hat{a}_n \).

B   SNR for Proposed Detection Technique

The photodetected signal and the local components are represented by Eqs. (25) to (28). The noise spectral density at the output of the PD, \( \Box \), is generally written as

\[
\eta = \frac{1}{4} \left[ e \mathcal{R} \left( \frac{P_{cl}}{2} + P_{c2} \right)
\right.
\]

\[
+ e I_D + \frac{2kT}{R_L} \right],
\]  (A11)

where \( \mathcal{R} \) \( = e \cdot \Box \) is the responsivity of the photodetector. The responsivity is assumed to be the white noise, which in order, consists of the shot noises for the signal and the local light, the dark current of the photodetector \( I_D \), and the thermal noise \( 2kT/R_L \). For the thermal noise, \( k \), \( T \), and \( R_L \) are the Boltzmann constant, the temperature, and the load resistance, respectively. When the shot noise is dominant, the noise spectral density, \( \Box \), is approximated as

\[
\eta \approx \frac{1}{4} e \mathcal{R} P_{c2} \left( 1 + \frac{1}{2} \frac{P_{cl}}{P_{c2}} \right).
\]  (A12)

\( s(t) \), \( l(t) \), and \( n(t) \) are assumed to be statistically independent of each other, and their averages are all zero. For the square-law detector in PNC, the self-correlation becomes
Here, \( E[\bullet] \) represents the ensemble average, and \( R_{x,y}(\cdot) \) is the correlation function of \( x(t+\tau) \) and \( y(t) \) and is defined as

\[
R_{x,y}(\tau) = E[x(t+\tau) \cdot y(t)].
\]

\( R_{x,x}(\cdot) \) is neglected because it is much smaller than \( 4 \{ R_{x,(\cdot)} \cdot R_{x,(\cdot)} \} \cdot R_{y,(\cdot)} \). In this case, our interest terms are the down-converted component, \( 4 \{ R_{x,(\cdot)} \cdot R_{x,(\cdot)} \} \cdot R_{y,(\cdot)} \), and the noise components, \( 4 \{ R_{x,(\cdot)} \cdot R_{x,(\cdot)} \} \cdot R_{n,n}(\cdot) \).

They become

\[
R_{x,x}(\tau) \cdot R_{y,y}(\tau) = \frac{1}{2} \left( \frac{1}{2} a_s a_l \right)^2 \cdot \frac{1}{2} \left[ \cos(2\pi f_{IF} \tau) \right] \left( 2 \Delta f + f_{RF} \right) \tau + \Delta \phi(t) \cdot R_{n,n}(\tau)
\]

where \( \delta(\tau) \) represents the delta function.

From the above calculation, under the shot-noise limited environment, the SNR for the proposed detection becomes

\[
\frac{S}{N} = \frac{\alpha P_{c1}}{B_{IF}} \frac{a_s^2 a_l^2}{a_s^2 + a_l^2} \cdot \frac{1}{4 \left( 1 + \frac{1}{B_{IF}} \right)} \cdot \frac{1}{2 \left( 1 + \frac{1}{P_{c1}} \right)} \cdot 2 \left( 1 + \frac{1}{P_{c1}} \right).
\]

Under the local shot-noise limited environment \( (P_c \geq P_{c1}) \), moreover, the theoretical limit of SNR is expressed as

\[
\frac{S}{N} \leq \frac{1}{2} \frac{\alpha P_{c1}}{B_{IF}} \frac{a_s^2 a_l^2}{a_s^2 + a_l^2} \cdot \frac{1}{2 \left( 1 + \frac{1}{P_{c1}} \right)} \cdot 2 \left( 1 + \frac{1}{P_{c1}} \right) \cdot \left( 1 + \frac{1}{P_{c1}} \right).
\]

The same manner, the SNR for PNC is derived and acquires the same result as the above SNR.

### C SNR for IM/DD

Two transmitting optical signals are assumed to have no data \( (\cdot) 0 \) and expressed using Eqs. (32) and (33), respectively. In this case, because the photocurrent is given by Eq. (34), the signal power, \( P_s \), becomes

\[
P_s = \frac{1}{2} l_s(t)^2 \cdot R_L
\]

\[
= 2 (R_g P_{c1} a_s a_l)^2 \cdot R_L.
\]

Here, \( G \) is the gain of an optical amplifier with a noise figure of \( n_{sp} \), and \( R_L \) is the load resistance. On the other hand, the noise from the photodetector is expressed as

\[
P_n = [e R g P_{c1} (a_s^2 + a_l^2) + e I_L + e \eta(G - 1) n_{sp} m_t \Delta f]
\]

\[
+ 2 e G (G - 1) n_{sp} R P_{c1} (a_s^2 + a_l^2)
\]

\[
+ e \eta(G - 1) n_{sp} m_t \Delta f + \frac{2 k T}{R_L}
\]

\[
\cdot 2 B_{IF} R_L.
\]

In the above equation, the components are in order the shot noise for the signal and local, the dark current, the amplified spontaneous emission (ASE) shot noise, the \( (signal + local) \) ASE term, the ASE ASE term, and the relative intensity noise (RIN), respectively. When the signal and local shot noises are dominant, \( P_s \) is approximated as

\[
P_n \approx 2 e G (G - 1) n_{sp} R P_{c1} (a_s^2 + a_l^2)
\]

\[
\cdot 2 B_{IF} R_L.
\]

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From the above, the SNR for the conventional IM/DD is approximated as

\[
\frac{S}{N} = \frac{P_s}{P_n} \approx \frac{\alpha P_{cl} \cdot a_0^2 a_1^2}{2n_{sp} \left( 1 - \frac{1}{G} \right) \left( a_0^2 + a_1^2 \right)}
\]  \hspace{1cm} (A23)

If the gain of the optical amplifier is large enough to suppress the other noise components \((G \gg 1)\), then the theoretical limit of SNR is given as

\[
\frac{S}{N} \leq \frac{1}{2n_{sp}} \cdot \frac{\alpha P_{cl} \cdot a_0^2 a_1^2}{B_{IF} \left( a_0^2 + a_1^2 \right)}. \hspace{1cm} (A24)
\]