2-3 Relativisitic Effects in Time and Frequency Standards

HOSOKAWA Mizuhiko

Recent technology on the precise measurement of time and frequency makes it rather easy to detect the relativistic effects. It is, therefore, indispensable to take these effects into account when we conduct such precise measurement. In this article, we will show the outline of the relativity and try an intimate illustration on most popular four relativistic effects; second Doppler effect, gravitational red shift, Sagnac effect and Shapiro delay.

Keywords
Relativity, Second doppler effect, Gravitational red shift, Sagnac effect, Shapiro delay

1 Introduction—Space-Time and the Lorentz Transformation—

One-dimensional time and three-dimensional space together constitute space-time. Under the theory of relativity, time and space are not independent, but instead should be treated as a unit. Current time and frequency standards are generated and compared with an accuracy in the order of approximately 10^{-15}. However, the relativistic effects caused by the motion of satellites or gravity near the Earth's surface are in the order of 10^{-10}, or over 10,000 times larger than the maximum precision of today's devices. Such discrepancies arising in the Newtonian concepts of space and time have been measured and verified definitively in a number of cases, relying on the increasing measurement precision of current time, space, and frequency standards. These relativistic effects cannot be ignored in today's wide range of high-precision time and frequency applications.

It is now essential to take into account the characteristics of four dimensional space-time (specifically, relativistic effects) in the precise treatment of time and frequency, both in global applications and in connection with today's space-related technologies. I have discussed the general aspects of the space-time reference frame earlier, in the 2000 journal[1].

Four main effects should be taken into consideration in the precise measurement of time and frequency: the second Doppler effect, the gravitational red shift, the Sagnac effect, and Shapiro delay. Here, I will try to describe these four relativistic effects as simply as possible, to provide an intuitive overview of the problems involved and of the concrete steps taken to address these problems.

First, I would like to clarify the basic rules of transformation. Coordinate transformation between two space-time reference frames in relative motion is expressed with time and space treated as a single quantity. In particular, when the relative velocity between the reference frames is constant and both systems can be considered inertial systems, the applicable transformation is the well-known Lorentz transformation. As the appendix of Reference[1] describes the derivation of this transformation in detail, I will provide only the results of this transformation here, as a basis for our treatment of relativity. Denoting the magnitude of the relative velocity between the reference frames as v, and assuming its direction is along the x-axis,

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx/c^2) \end{aligned} \qquad (1)$$

Here, c is the velocity of light in a vacuum, 3×10^8 m/s, and γ is the frequently used constant, defined as follows:

$$\gamma \equiv 1/[1-(v^2/c^2)]^{1/2} \qquad (2)$$

When space-time is expressed on two-dimensional paper, one of the space axes is chosen as the representative and is combined with the time axis. Here, let us choose the x-axis as the representative. First, establish a reference frame in space-time represented by a certain time axis (t-axis) and space axis (x-axis). Next, draw a new reference frame connected to this reference frame by the Lorentz transformation and represent it by a t'-axis and an x'-axis. Fig.1 shows the relationship between these two reference frames. If the reference time clock is stationary at the origin of the space of the first frame, it is understood that the trajectory of this clock in space-time (referred to as the "world line") becomes the time axis. Then, the reference frame connected to the first reference frame by the Lorentz transformation gains a new time axis, formed by the clock moving at a constant velocity in the original reference frame. It is also understood that the new space axis (x'-axis) is transformed as shown in Fig.1 due to the principle of the constant speed of light. A given point in space-time is designated differently in the different reference frames. The transformation expressed in Eq.1 provides the rule for conversion between these designations.

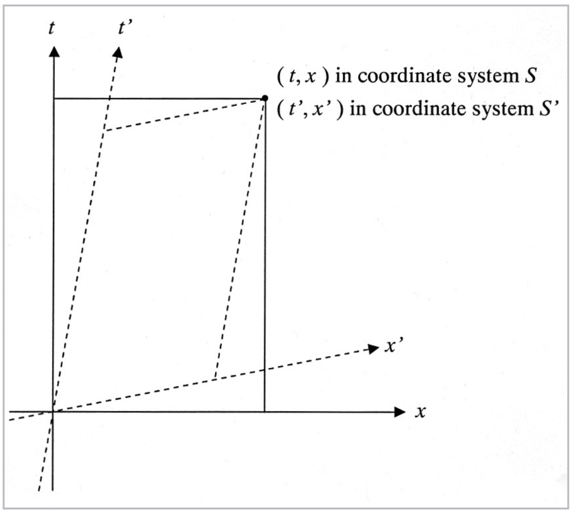

Fig.1 Space-time and the Lorentz transformation of the coordinate axes

2 Invariants and the Minkowski space: the second Doppler effect

When a value obtained in a reference frame does not change in a transformation, the value is referred to as an invariant under the transformation. For example, in normal Euclidean space, the components of the vector connecting any two points change under this rotational transformation; however, the sum of the squares of the components (the square of the length of the vector) remains unchanged, in accordance with the Pythagorean theorem. Thus, the vector length (and its square) is an invariant in coordinate rotation. Conversely, when one considers a transformation in which the sum of the squares of the vector components is an invariant, rotation is obtained as the solution. Determining precisely which values become invariants is thus extremely important in any attempt to define the nature of space.

Four-dimensional space-time, in which special relativity holds, has a unique invariant, as described below, that is an equivalent to the Minkowski space in mathematics. Choose two arbitrary space-time points (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2) in an inertial reference frame. Assume that these two points are expressed as (t'_1, x'_1, y'_1, z'_1) and (t'_2, x'_2, y'_2, z'_2), respectively, in another reference frame, connected to the first by the Lorentz transformation. Under the special theory of relativity, the values of the space-time coordinates themselves and the differences between the components change under the Lorentz transformation (for example, $t_2-t_1 \neq t'_2-t'_1$). However, it can be shown using Equation (1) that the Lorentz transformation does not change the sum of the squares of the differences between the components:

$$\begin{aligned} &c^2(t_2-t_1)^2-(x_2-x_1)^2-(y_2-y_1)^2-(z_2-z_1)^2 \\ &=c^2(t'_2-t'_1)^2-(x'_2-x'_1)^2-(y'_2-y'_1)^2-(z'_2-z'_1)^2 \end{aligned} \qquad (3)$$

Substituting Equation (1) in the right-hand side of the above equation and rearranging the terms results in the expression given on the left-hand side of the equation (it is recommended that the reader perform this operation once, for reference). Equation (3) holds for any two arbitrary points. However, we will restrict ourselves to a point (t, x, y, z) and its vicinity (t+dt, x+dx, y+dy, z+dz). The local characteristics of the infinitesimal line element at an arbitrary position can then be expressed as:

$$c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 = c^2\mathrm{d}t'^2 - \mathrm{d}x'^2 - \mathrm{d}y'^2 - \mathrm{d}z'^2 \qquad (3')$$

Equation (3') is equivalent to Equation (3) under the special theory of relativity, except that it is expressed in local form. Later it will become clear that Equation (3') is a more convenient expression in terms of the general theory of relativity.

We now show that the second Doppler effect, the time dilatation of a moving body, is calculated using Equation (3'). Consider a clock moving in a reference frame indicated by numbers without dashes. The motion could be in any direction, but for the sake of simplicity, let us define the axes such that the direction of motion is along the x-axis at velocity v. The displacement during infinitesimal time dt is expressed as vdt. The vector of motion in space-time is expressed as (dt, vdt, 0, 0). Now, choose the reference frame in which the motion of the clock during this period appears stationary and denote it by using numbers with dashes: in other words, choose the reference frame that moves with the clock as the reference frame by using numbers with dashes. The vector of motion is now expressed as (dt', 0, 0, 0) in this reference frame. Here, dt' is the proper time of the moving clock.

As the value given in Equation (3') does not change between the two reference frames, the following is obtained:

$$c^2\mathrm{d}t^2 - v^2\mathrm{d}t^2 = c^2\mathrm{d}t'^2 \qquad (4)$$

Thus, the relationship between the reading of the moving clock and coordinate time is:

$$\mathrm{d}t = \mathrm{d}t'/(1 - v^2/c^2)^{1/2} \qquad (5)$$

This relationship holds only for the short period of time during which velocity may be considered constant. In general, dt' should be replaced by the progression of proper time dτ Integrating Equation (5) with changing velocity gives

$$t = \int \mathrm{d}\tau /(1 - v(\tau)^2/c^2)^{1/2} \qquad (6)$$

This is known as "the second Doppler effect" under the special theory of relativity.

3 Inner products and the metric tensor

Each side of Equation (3) differs in sign from the Euclidean invariant "square length. In Euclidean space, "square length" can be expressed as follows, using a row vector and a column vector:

$$\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 = (\mathrm{d}x, \mathrm{d}y, \mathrm{d}z)\begin{pmatrix}\mathrm{d}x\\ \mathrm{d}y\\ \mathrm{d}z\end{pmatrix} \qquad (7)$$

The left-hand side of Equation (3) can be expressed similarly as

$$c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 = (c\mathrm{d}t, \mathrm{d}x, \mathrm{d}y, \mathrm{d}z)\begin{pmatrix}1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}\begin{pmatrix}c\mathrm{d}t\\ \mathrm{d}x\\ \mathrm{d}y\\ \mathrm{d}z\end{pmatrix} \qquad (8)$$

The left-hand side of Equation (8) is denoted as ds^2. Here, ds indicates an infinitesimal line element and ds^2 is its square length in Minkowski space. With reference to Equation (8), Equation (7) can similarly be expressed as follows:

$$\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 = (\mathrm{d}x, \mathrm{d}y, \mathrm{d}z)\begin{pmatrix}1 & & \\ & 1 & \\ & & 1\end{pmatrix}\begin{pmatrix}\mathrm{d}x\\ \mathrm{d}y\\ \mathrm{d}z\end{pmatrix} \qquad (7')$$

Here, the square matrices are all diagonal matrices and their non-diagonal components are expressed by blanks when they are 0. Detailed discussion of this topic is more appropriately reserved for mathematics textbooks; here we may simply state that the inner product is the product of the row vector and the transposed column vector, but only in Euclidean space. Generally speaking, a matrix describing the nature of the space must be inserted between the vectors. This matrix, which appears in the inner product of the vectors and expresses the nature of the space, is known as the metric tensor. Tensor is the extended notion of the vector: scalars are zero-order tensors, and vectors are first-order tensors; tensors of higher orders can also be considered, as required. The metric tensor is a second-order tensor and can be expressed in equations as a square matrix. The general expressions for higher-order tensors involve required numbers of subscripts[2]-[4], but here we will restrict our discussion to tensors of up to the second order; we will express these tensors using matrices, which are intuitively easier to understand. Further, although metric tensors generally have non-zero non-diagonal components, here we will limit our discussion to cases in which this degree of complexity does not arise.

The Euclidean metric tensor need not be inserted explicitly; that is, it is expressed with the unit matrix. On the other hand, the Minkowski metric tensor is expressed as a matrix with space and time having opposite signs. Generally, the square length ds^2 of the line element, which is an invariant under the coordinate transformation, is expressed as follows:

ds^2 = (row vector of the line element)
(metric tensor)
(column vector of the line element). (9)

These expressions may seem overly formal and of limited use. Certainly, when each component of the metric tensor is expressed as a constant, an expression such as Equation (3) seems sufficient, with no need for expressions such as Equation (8). However, using the metric tensor provides a variety of potential advantages in comprehending the nature of space-time and in performing associated calculations. Furthermore, the metric tensor is indispensable when discussing general theory of relativity's treatment of the curvature of space-time. If a metric tensor is provided for an all-encompassing space-time continuum, the relationship between coordinate time (or each component of coordinate time) and the proper time or the proper length may be obtained using this tensor. It will then be possible, for example, to compare time and frequency measurements at one position with those at another position, or to establish a coordinate time that can provide wide-ranging synchronization.

The details of the metric tensor are quite complicated, so let us concentrate on obtaining a qualitative understanding, until we establish a conclusive equation for relativistic effects.

To say that the value of the metric tensor is different at every point in space is equivalent to saying that space is curved. Here it may be helpful to consider an analogous ordinary curvature. The curvature of a given curved space is defined by the second derivative of the metric tensor with respect to position. To say that the space is "flat" is to say that the second derivative of the components of the metric tensor of the space is zero everywhere: in other words, the metric tensor is constant throughout the space. Further, expressing the metric tensor as in Equation (8) means that the space-time is flat, that all space axes are orthogonal, uniform, and isotropic, and that the reference frame is an inertial reference frame that does not rotate with time.

Einstein had the insight that gravity is the result of the warping of four-dimensional space-time, constructing his general theory of relativity based on this conclusion[2]-[4]. Newton's law of universal gravitation states that all bodies receive equal acceleration in the gravitational field, while other theories concerning curved space (including the general theory of relativity) state that the space-time common to such bodies is itself curved.

The law describing the curvature of space-time, elucidated by Einstein within the general theory of relativity, may be qualitatively expressed as an equation in the following form:

curvature of space-time = distribution of energy and momentum (10)

It has been demonstrated in various experiments and observations that this equation describes the laws of gravity in our world more precisely than Newton's universal law of gravity[5]. If one obtains the metric tensor for all of space as the solution to Equation (10), the practical handling of space-time may be definitively established for applications relating to time and frequency standards.

More than one theory describes curved space-time: apart from Einstein's general theory of relativity, the Brans-Dicke theory[5] is a well-known example. However, observation results to date support the general theory of relativity. Adjusting the arbitrary parameters of theories such as the Brans-Dicke theory can produce results that do not contradict the general theory of relativity, but the fact remains that no space-time phenomena have been observed that cannot be explained based on the general theory of relativity[5]. Thus, here we will take only the general theory of relativity as our basic theory.

4 Approximate solution of Einstein's equation

It is extremely complicated to express Equation (10) quantitatively, and exceedingly difficult to obtain an exact solution. In fact, exact solutions to this equation have been obtained only in a few cases, each featuring restricted conditions. However, an approximate solution is available that can supply sufficient precision for use in the weak gravitational fields found in the solar system, although it cannot be applied to extremely strong gravitational fields, such as those found in the vicinity of a black hole. The expression of the solution is not complicated for a non-rotating reference frame:

$$ds^2 = (cdt, dx, dy, dz)\begin{pmatrix} 1+2\Phi/c^2 & & & \\ & -1+2\Phi/c^2 & & \\ & & -1+2\Phi/c^2 & \\ & & & -1+2\Phi/c^2 \end{pmatrix}\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} \quad (11)$$

Here, $\Phi<0$ is Newton's gravitational potential, which varies at each position in space-time, depending on the distribution of matter. Whether the employed reference frame is rotating or non-rotating can be determined mechanically based on the presence or absence of apparent forces (such as centrifugal force) acting on a stationary body within the reference frame. In the reference frame of the surface of the Earth, the centrifugal force and Coriolis forces are observed; it can therefore be assumed that we are dealing with a rotating reference frame. Conversely, in a non-rotating reference frame in which Equation (11) holds, the Earth's surface appears to rotate while the surface of the celestial sphere appears stationary.

The approximate solution (11) can be considered to hold up to a frequency precision of approximately 10^{-17} when considering the distance between the center of the Earth and general satellite orbits, and up to approximately 10^{-15} when considering greater distances within the solar system (with the exception of locations extremely close to the sun). Thus, this approximation is capable of providing sufficient precision for current methods of generation and measurement of time and frequency standards. Nevertheless, in consideration of recent and anticipated improvements in the precision of these standards, more precise approximate solutions involving the metric tensor have recently come under discussion [6].

In Equation (11), a position infinitely far from all masses is chosen as the reference point at which the metric tensor becomes equal to the reference point for the Minkowski space. In the Terrestrial reference frame fixed to the Earth surface, we chose the geoid surface, a gravitational equipotential surface, as the practical reference point for the frequency standard. With this approach, Equation (11) must be modified:

$$ds^2 = (cdt, dx, dy, dz)\begin{pmatrix} 1+2\Phi'/c^2 & & & \\ & -1+2\Phi'/c^2 & & \\ & & -1+2\Phi'/c^2 & \\ & & & -1+2\Phi'/c^2 \end{pmatrix}\begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix} \quad (11')$$

Here, Φ' is the gravitational potential with respect to the geoid surface, and can be expressed with potential U at the geoid surface as follows:

$$\Phi' = \Phi - U \qquad (12)$$

It should be noted here that subtracting the potential U at the geoid surface induces a scale transformation on the time coordinate axis. This scale transformation causes a difference in clock rate between Terrestrial Time (TT), defined with respect to the proper time on the geoid surface, and Geocentric Coordinate Time (TCG), defined with respect to the proper time at an infinitely remote position. The shift of the value away from 1 is expressed by the value LG = 6.9693×10^{-10}. See References [1] and [4] for the definition and transformation of these time scales.

5 Gravitational red shift

In the Terrestrial reference frame, Φ' near the Earth's surface can be approximated using gravitational acceleration g = 9.8 m/s^2 and height h from the geoid surface:

$$\Phi' = +gh \qquad (13)$$

With the above value for g, the correction term $2\Phi'/c^2$ for the metric tensor changes by 2.2×10^{-16} per meter. Let us consider a stationary body near the surface of the Earth. Here, $2\Phi'/c^2 << 1$ holds. Further, the relation dx = dy = dz = 0 also holds, because the position of the body does not change after time dt. The proper time of this body during this period is expressed as follows:

$$d\tau = ds/c = (1+2\Phi'/c^2)^{1/2}dt \approx (1+\Phi'/c^2)dt \qquad (14)$$

At a position higher than the geoid surface, $\Phi'<0$ holds; thus the higher the position of the body, the shorter the progression of coordinate time during the same period of proper time: in other words, proper time elapses at a faster rate. It can be shown that as the gravitational potential decreases—that is, as its absolute value increases with respect to the infinitely remote zero potential point of gravity in Equation (11)—time passes more slowly. This phenomenon is known as the gravitational red shift. As Equation (14) and the value $2\Phi'/c^2 = 2.2\times10^{-16}$/m indicate, the frequency for the proper time of a body near the Earth's surface increases by 1.1×10^{-16} when the height of the body increases by 1 m.

When a body is moving, the second Doppler effect, expressed by Equation (5), is added to this effect. This can easily be derived by performing calculation under the assumption that dx, dy, and dz are not zero. Here it should be noted that as the spatial parts of the metric tensor is divided by c^2 in the course of calculation, the effect of the shift away from 1 is extremely small, in the order of $1/c^4$. Approximating up to the order of $1/c^2$, the following expression is obtained:

$$d\tau = (1+\Phi'/c^2 - v^2/2c^2)dt \qquad (15)$$

The proper time of a moving body can be obtained in this manner, taking the general theory of relativity into account in calculation. In particular, in the case of a satellite following an elliptical orbit, velocity increases at the perigee (where the potential is small), which slows the proper time at the perigee even further, due to the reinforcement of the two effects, while the reverse occurs at the apogee. This reinforcement of the second Doppler effect and the gravitational red shift is known as the eccentricity effect. For circular orbits, Equation (15) can be simplified and an orbit referred to as a time-geostationary orbit can be assumed for terrestrial time TT, which is defined with respect to atomic time on the geoid surface[7][8].

6 Rotating reference frames and the Sagnac effect

As has been discussed, a body fixed to the Earth's surface is rotating in a non-rotating reference frame because the Earth is rotating. However, almost all of the atomic clocks that determine the standards of time and frequency (including all of the clocks contributing to International Atomic Time, or TAI) are fixed to the Earth's surface. Thus when considering the synchronization of these surface clocks, we must take the Sagnac effect into account, a relativistic effect that poses a number of vexing problems.

One can determine whether the reference frame is rotating or non-rotating based on the presence or absence of centrifugal force on a stationary body within the reference frame. Ignoring the effects of gravity, light in a vacuum travels at a constant velocity and in a straight line only in a non-rotating reference frame. In a rotating reference frame, light travels in a helical path, depending on the direction of the rotation. This causes the Sagnac effect, which only occurs within a rotating reference frame. It is best to use the metric tensor of a rotating reference frame for the general treatment of the Sagnac effect for wide areas, but here we will first discuss this effect in a local sense, using the special theory of relativity.

Let us take a point on the equator as the origin and consider two reference frames at a given instant. One is a stationary reference frame designated as Reference Frame A, and is independent of the motion of the Earth. The other reference frame, Reference Frame B, moves at a constant velocity in the direction and at the speed of the rotation of the origin at that instant. The speed of the Earth's rotation at the equator is approximately 450 m/s. The point chosen as the origin at the given instant moves in Reference Frame A at this speed of rotation, but can be considered to remain at the origin in Reference Frame B for a short period of time, during which the effect of rotation is negligible. As shown in Fig.2 and in the Lorentz transformation (1), the two times for the same space-time point diverge as the distance from the origin in Reference Frame A increases. Obviously Reference Frame A can more broadly express time and space on the Earth, because the Earth is rotating with respect to the inertial reference frame. The preparation and linkage of additional reference frames (such as Reference Frame B) for multiple points on the equator will result in multiple time values—i.e., a different time for each reference frame (Fig.3).

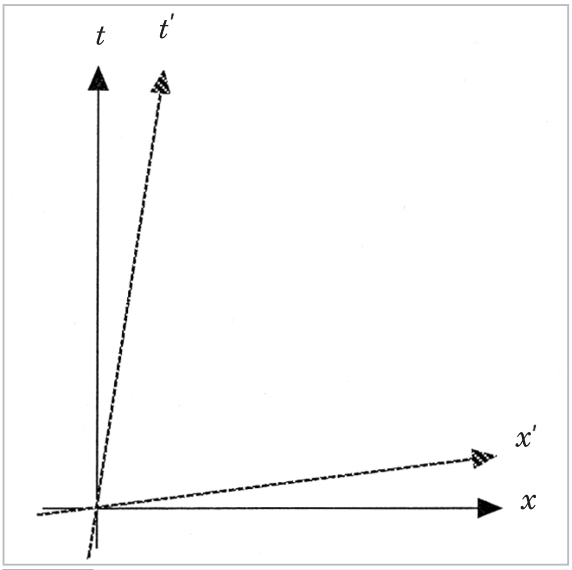

Fig.2 *Inertial reference frame and a reference frame that follows the rotation of the Earth*

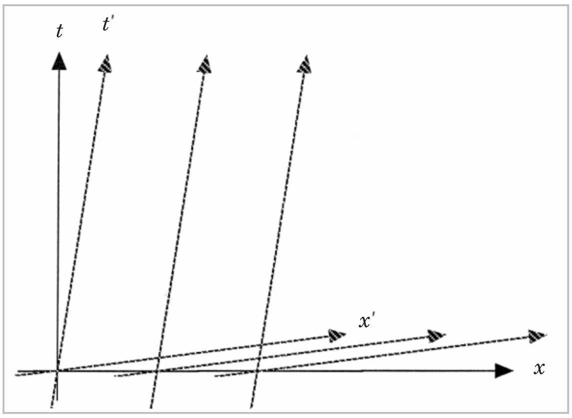

Fig.3 *Linkage of reference frames following the rotation of the Earth*

If these frames were to be linked and time shifted to force consistency among the frames, time after one rotation will differ significantly from the original time. It is clear that there is no way of preparing rotating reference frames that are both locally inertial and capable of sharing common time over a wide range. This is due to the phenomenon in which the speed of eastbound light decreases by the speed of rotation and the speed of westbound light increases by the same amount within a reference frame moving along with the rotation of the Earth. This phenomenon is a logical consequence of the constant velocity of light within a non-rotating reference frame (independent of the eastbound rotation of the Earth). This effect is referred to as the Sagnac effect, after the scientist who discovered it. Reference [9] is a well-known work describing the appearance of this effect in international time comparison.

The Sagnac effect poses a significant problem in the establishment of time standards using instruments fixed on the surface of the Earth. Two methods can be used to resolve this problem: the definition of the propagation velocity of light could be modified to allow for different speeds for eastbound light and westbound light, or a correction term could be introduced to describe the time delay for a point in the east relative to another point in the west. The latter method is employed in practice. The correction term Δt for distance x in the east-west direction is expressed as follows:

$$\Delta t = \Omega_E R x / c^2 \qquad (16)$$

Here, Ω_E is the angular velocity of the Earth, R is the radius of the Earth, x/c is the time required for light to travel distance x, $\Omega_E Rx/c$ is the distance the Earth surface moves during this period of time, and Δt is the time required for light to catch up. A more general treatment of rotating reference frames in time comparison using the metric tensor is described in detail in Reference[10]. The metric tensor of a rotating reference frame has non-zero non-diagonal components; accordingly its treatment is more complicated.

7 Shapiro delay

In the theory of curved space-time, the motion of light in a vacuum is determined based on the assumption that the length given in Equation (11) for the line element ds (the motion of light) of the world line is zero everywhere. This corresponds to the conclusion that space can be locally approximated as Minkowski space and that a coordinate transformation may be obtained in which the special theory of relativity holds. This in turn indicates that the special theory of relativity does not always hold globally and that light does not always travel at a constant speed in curved space-time. Let us consider the simple example of light traveling along the x-axis with a mass M at the origin, using the approximated solution (11) of the general theory of relativity. Here, $y = z = 0$ and $\Phi = -GM/x$ always hold; thus the metric tensor is simplified to a 2×2 matrix:

$$ds^2 = (c\,dt, dx)\begin{pmatrix} 1-2GM/xc^2 & \\ & -1-2GM/xc^2 \end{pmatrix}\begin{pmatrix} c\,dt \\ dx \end{pmatrix} \qquad (17)$$

Here, G is Newton's gravitational constant, 6.67×10^{-11} $m^3/(kgs^2)$. The motion of light is obtained by expanding this expression and substituting $ds^2 = 0$:

$$(1-2GM/xc^2)c^2dt^2 = (1+2GM/xc^2)dx^2 \qquad (18)$$

From this equation and the approximation $1 >> 2GM/xc^2$ for a weak gravitational field, the velocity of light dx/dt at point (t, x) in space-time within this reference frame is smaller than the so-called velocity of light c:

$$dx/dt = \pm(1-2GM/xc^2)c \qquad (19)$$

This equation clearly shows that the velocity of light is zero at $x = 2GM/c^2$, which is separated from the origin by the Schwarzschild radius; in this case the distance from the origin is small and the gravitational field is strong. The equation also shows that the velocity of light equals the normal value c at the far limit, where x is large. Note also that velocity does not depend on the direction of the motion of light; i.e., on whether it is traveling toward or away from mass M, the source of the gravitational field.

As demonstrated in this example, in a reference frame that satisfies the general theory of relativity in a wide range, the velocity of light becomes smaller where the gravitational field is stronger, and as a result, it is deduced that the propagation time of light within the gravitational field will appear to decrease relative to that obtained assuming velocity *c*. I. Shapiro first described this effect in a paper submitted to a referred journal[5][11]. This effect is thus referred to as Shapiro delay. In his paper, Shapiro demonstrated that the propagation of light or electromagnetic waves between planets shows delay when calculated within the framework of the general theory of relativity, relative to results of calculation that do not take the general theory of relativity into account. Until then, only three experiments had been proposed to verify the general theory of relativity: one dealing with the movement of perihelion of Mercury, the gravitational lens effect, and the gravitational red shift. Thus Shapiro entitled his paper the "Fourth Test of the General Theory of Relativity." This effect was observed in radar-echo experiments involving Mercury and Venus and played a significant role in the verification of the general theory of relativity[5].

The delay in propagation time can easily be calculated, using Equation (19), on an axis that passes through the gravity source, as in the example above. Considering the two points x_1 and x_2 in the positive region on the x-axis, the definite integral of

$$c\mathrm{d}t = (1 + 2GM/xc^2)\mathrm{d}x \qquad (20)$$

yields delay Δt during actual propagation of light from x_1 to x_2, relative to propagation assuming velocity c:

$$\Delta t = 2GM/c^3 \ln(x_2/x_1) \qquad (21)$$

For general propagation in three-dimensional space using a post-Galilei's approximation of sufficient precision, based on the length of the sides of the triangle made by x_1, x_2, and the position of the gravity source P (Fig.4), Δt can be obtained by solving the following expression:

$$\Delta t = 2GM/c \ln\{(|Px_1| + |Px_2| + |x_2x_1|)/(|Px_1| + |Px_2| - |x_2x_1|)\} \qquad (22)$$

As an example, let us assume that x_1 is the radius of revolution of Venus (0.7 astronomical units), x_2 is the radius of revolution of the

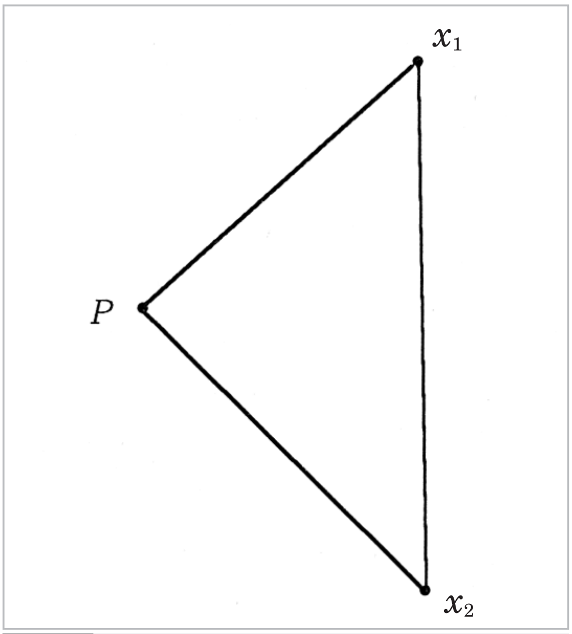

Fig.4 Shapiro delay: triangle made by the initial and final points and the gravity source

Earth (1 astronomical unit), and M is the mass of the Sun, 2×10^{30} kg. In this case the Shapiro delay for the propagation of light between Venus and the Earth at the inferior conjunction of Venus when the Sun, Venus, and the Earth are arranged in a line is approximately 3.5 microseconds one way, and 7 microseconds for the round trip.

The elongation of the light path attributable to bending due to the gravitational lens effect is negligible compared to the Shapiro delay, as long as the bending angle is not large. As the bending angle resulting from the gravitational lens effect of the Sun is less than 2 arc-seconds (even where this effect is greatest, where the light grazes the edge of the sun) such bending may be deemed negligible within the solar system.

8 Conclusion

Using the Lorentz transformation, we have introduced the metric tensor as an invariant and have outlined four relativistic effects: the second Doppler effect, the gravitational red shift, the Sagnac effect, and Shapiro delay. These effects all seem strange at first, but become intuitive and easy to understand once the concept of space-time is grasped. In this sense, I hope that this article will prove of some assistance to the reader's understanding of the nature of space-time. I could not present a number of more detailed treatments and more general expressions here; please see the books listed in the reference list for further information. I hope to provide more detailed quantitative treatments and descriptions, in later reports. In the meantime, the reader is recommended to consult available articles on the influence of relativistic effects on the GPS [12], which provide useful discussions of the subject at hand.

I hope that this report will provide an intuitive understanding of relativistic concepts and of the four effects discussed above, and that it will assist in the evaluation of the type and amount of practical effects, particularly in activities such as time and frequency measurement or time comparison.

References

1 M.Hosokawa, Review of the Communications Research Laboratory, Time scales and Frequency Standards Special issue, Vol.45, Nos.1/2, pp.3, 1999. (in Japanese)

2 A.Einstein, S. B. Preuss.Akad. Wiss., pp.777-786, pp.799-801 , 1915.

3 L.D.Landau, E.M.Lifshiz, "The Classical Theory of Fields" §6, Butterworth-Heinemann; 4th edition. 1987.

4 T.Hukushima, "Gendai Sokuchigaku", §3, pp.105-154, Bumkenshya, 1994. (in Japanese)

5 Clifford M.Will, "Was Einstein Right? Putting General Relativity to the Test", Second edition published by Basic Books, New York, 1993.

6 IAU Colloquium 180 Resolutions, 30 March 2000. http://danof.obspm.fr/IAU_resolutions/Resol-UAI.htm

7 M.Hosokawa and F.Takahashi, Publ. Astron. Soc. Japan 44, pp.159-162 , 1992.

8 M.Hosokawa, F.Takahashi, and M.Yoshikawa, Review of the Communications Research Laboratory, Time scales and Frequency Standards Special issue, Vol.45,, Nos.1/2, pp.103-, 1999. (in Japanese)

9 Y.Saburi, M.Yamamoto, and K.Harada, IEEE Trans. Instrum. Meas., IM-25 No4, pp.473 - 477, 1976.

10 G.Petit and P.Wolf, Astron.Astrophys. 286, pp.971-977, 1994.

11 I. Shapiro, Phys. Rev. Lett. 13, pp.789-, 1964.

12 N.Ashby, M.Hosokawa, Parity Jun. pp.18-, Maruzen Co.Ltd., Tokyo, 2003.

***HOSOKAWA Mizuhiko**, Ph. D.*

Leader, Atomic Frequency Standards Group, Applied Research and Standards Division

Atomic Frequency Standards, Space Time Measurements