3-2-3 The Improvement of Frequency Stability Using the Collimation Apparatus of the Launched Atomic Fountain

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We describe a new method of reducing the horizontal velocity components of launched atoms using a laser beam in the vertical direction. This laser beam produces a harmonic potential in the horizontal direction. If the initial atomic position is close to the center of the laser beam, the horizontal velocity component becomes almost zero after a quarter period of harmonic oscillation. If the laser is switched off at this moment, the horizontal velocity component is very small afterwards. This method is expected to be useful to improve the S/N ratio of the spectrum observed with the atomic fountain frequency standard.

Keywords
Atomic fountain, Collimation, Dipole force, Harmonic potential, Phase space volume

1 Introduction

We are currently developing an atomic-fountain type frequency standard whereby the Ramsey signal is observed by launching cesium atoms (cooled to approximately a few μK by the laser cooling method) in the vertical direction such that they pass through a microwave resonator twice in the process of rising and falling (refer to Section **3-2-2**). The signal-to-noise ratio (S/N ratio) of the signal obtained is limited by the degree to which the atomic fountain diffuses in the horizontal direction after launch. If the atoms are cooled to approximately 2 μK, the velocity component becomes about 1.6 cm/s on average, which means that the atoms will spread to a diameter of about 3.2 cm in one second[1]. If the hole of the microwave resonator is assumed to be 1 cm in diameter, it follows that the atoms that interfere with the microwave twice and contribute to the Ramsey signal make up only about 1/10 of the launched atoms. It is expected that if the horizontal velocity component can be reduced down to the range of a few mm/s, S/N ratio will increase considerably, leading to improved frequency stability. Therefore, several trials are underway in which the horizontal velocity component is reduced by Raman cooling; however, these experiments require complex operations, and the two-dimensional cooling effect has yet to be achieved[2]-[4].

The laser-cooling mechanisms established thus far have all relied on scattering force. "Scattering force" refers to the force to which the atom is subjected when it absorbs or emits a photon. Since the scattering force is not a conservative force, it is possible to make the phase space volume quite small. However, as the atom receives a recoil force in a random direction at the time of spontaneous emission, it is impossible to attain a temperature below the energy of one photon recoil relying solely on the scattering force. The velocity selective coherent population trap (VSCPT)[5] and Raman cooling[4] were developed to obtain a temperature below this limit, and were designed to prevent atoms having zero velocity or near-zero velocities from being subject to the scattering force.

In addition, the atom is subject forces from light, including the dipole force. This is the interaction between an optical electric field and the dipole moment of the atom induced thereby. Since this is a conservative force, it cannot change the phase space volume. Therefore, it has been used up to now mainly for trapping cryogenic atoms. In the initial phase, lasers of frequencies close to the resonance frequency had been used[6], but lately laser light of a frequency deviating significantly from the resonance is often used[7][8]. If the frequency deviates significantly from the resonance, a large laser power density is needed to obtain sufficient trap potential. Such laser light is convenient for obtaining a high-density trap because the heating effect by scattering is negligible.

In this paper, we propose and analyze a method of reducing the horizontal velocity component of the atomic fountain using the dipole force received from laser light. It is impossible to change the phase space volume using the dipole force, but it is possible to make the momentum distribution (velocity distribution) small while extending the positional distribution. If irradiation of laser light is stopped when the velocity distribution reaches a minimum, the minimum velocity distribution will subsequently be maintained. If non-resonant laser light is used, the influence of scattering can be ignored, and it becomes possible to attain a temperature below the energy of one photon recoil.

2 Analysis under the one-dimensional model

The potential $U(x,y,z)$ obtained by the dipole force that an atom receives from light is expressed by

$$U(x,y,z) = -\frac{1}{2}\alpha E_0^2 I(x,y,z) \tag{1}$$

where α denotes atomic polarizability and can be approximated by a value in a DC electric field assuming that the laser light is in a non-resonant infrared region (in the case of Cs atom, 59.6 A^3). $I(x,y,z)$ denotes power density distribution in the case where the direction of propagation of light lies on the z-axis, and its value at the origin is set to $I(0,0,0) = 1$. E_0 is the amplitude of the optical electric field at $(x,y,z) = (0,0,0)$. In this chapter, we consider the case of

$$I(x,y,z) = \exp\left[-\frac{x^2}{(\Delta x)^2}\right] \tag{2}$$

Here, Δx is a parameter that indicates the degree of broadening of the power density distribution of a laser. The kinetic equation of the atom is then expressed by

$$\frac{d^2x}{dt^2} = -\omega^2 x \exp\left[-\frac{x^2}{(\Delta x)^2}\right]$$

$$\omega^2 = \frac{\alpha E_0^2}{M(\Delta x)^2} \tag{3}$$

where M denotes the mass of the atom. If $x << \Delta x$ is satisfied, equation (3) is approximated by

$$\frac{d^2x}{dt^2} = -\omega^2 x \tag{4}$$

Here, if the laser light is irradiated only for a time of $0 < t < T$ and is not irradiated for a time of $T < t$, temporal variations of the position and velocity components in the x-direction (v_x) are expressed by

$$\begin{aligned} v_x(t) &= v_x(0)\cos\omega t - x(0)\omega\sin\omega t && 0<t<T \\ &= v_x(T) && T<t \\ x(t) &= x(0)\cos\omega t + \frac{v_x(0)}{\omega}\sin\omega t && 0<t<T \\ &= x(T) + v_x(T)(t-T) && T<t \end{aligned} \tag{5}$$

Here, setting $T = \pi/2\omega$, $v_x(T)$ is reduced to $-x(0)\omega$ regardless of the value of $v_x(0)$. If the initial position is at the center of the laser beam (i.e., $x(0) = 0$), the velocity component in the x-direction will become exactly zero for $t > T$, independent of the initial velocity. Fig.1 shows the trajectory of an atom when its initial position is set as $x(0) = 0$. In reality, not all of the initial positions of the atoms are

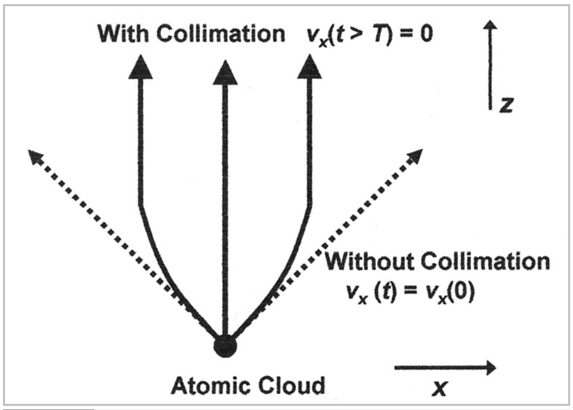

Fig.1 The trajectories of atoms after launch both with the collimation described in this paper and without the collimation, respectively. Here, the initial positions of the atoms are assumed to be at the center of the laser beam.

located exactly at the center of the laser beam, and atoms whose initial positions are off the center of the laser beam may be accelerated. Therefore, the temperature that represents the distribution of $v_x(t>T)$ will depend on the width of the distribution of $x(0)$. Here, consider one example assuming

$$\begin{aligned} \Delta x &= 1.5 \ \text{mm} \\ T &= 0.1 \ \text{s} \ (\omega = 2\pi \times 2.5 \ \text{radian/s}) \end{aligned} \tag{6}$$

When Cs atoms satisfy this condition, the power density at x = 0 becomes 25.4 W/cm^2 (E_0 = 1.38 × 10^5 V/m). If $x(0)$ is set to 0.5 mm, $v_x(T)$ will become 7.8 mm/s. However, this estimate was derived by approximating equation (3) with equation (4). In reality, this is effective only when x always satisfies |x| << Δx within a time of $0 < t < T$. Fig.2 shows

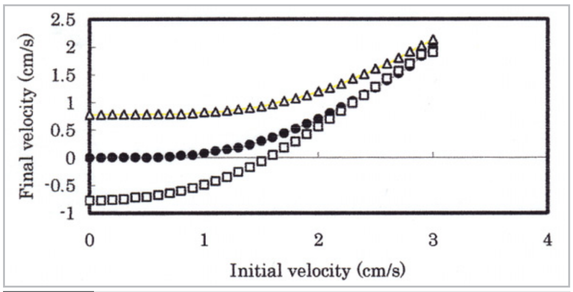

Fig.2 The final velocity of the atom, as a function of initial velocity, after being irradiated by the collimation laser under the conditions of equation (6). Symbol • represents the case in which x(0) = 0 mm; △ the case in which x(0) = −0.5 mm; and □ the case in which x(0) = 0.5 mm, respectively.

values of $v_x(T)$ calculated from the equation (3) as a function of the initial velocity for three different values of $x(0)$: −0.5 mm, 0 mm, and +0.5 mm. The estimate by $v_x(T) = -x(0)\omega$ is true for $v_x(0) < 1$ cm/s. It turns out that if the temperature corresponding to the distribution of $v_x(0)$ is 3 μK or lower and the distribution of $x(0)$ is in a range of ±0.5 mm, the distribution of $v_x(T)$ corresponds to a low temperature that cannot be reached by polarization gradient cooling.

In order to estimate in practice a temperature corresponding to the distribution of $v_x(t>T)$, assume that both initial velocity and position are Gaussian distributions, as described below. Here, $\delta_v(0)$ and $\delta_x(0)$ are parameters that indicate degrees of broadening of the distributions of initial velocity and of initial position, respectively, as shown in the following equation.

$$\rho_{v_x(0)} = \frac{1}{\delta v\sqrt{\pi}} \exp\left[-\frac{v_x(0)^2}{(\delta v)^2}\right]$$

$$\rho_{x(0)} = \frac{1}{\delta x\sqrt{\pi}} \exp\left[-\frac{x(0)^2}{(\delta x)^2}\right] \tag{7}$$

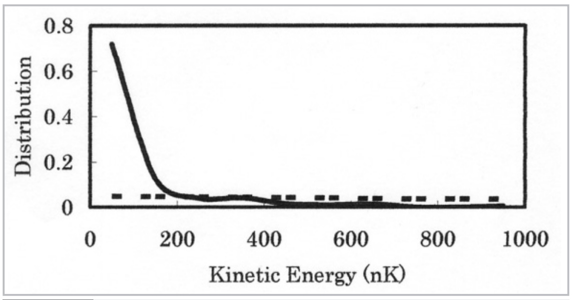

Fig.3 The solid line shows the kinetic energy distribution after irradiation by the collimation laser light under the conditions of the equations (6), (7) (one dimensional). The dotted line shows the kinetic energy distribution in the initial state (in thermal equilibrium at 2.6 μK).

Fig.3 shows the kinetic energy distribution of the atom after being irradiated by non-resonant infrared laser light under the conditions of equation (6), in the case of $\delta_v = 1.8$ cm/s (equivalent to 2.6 μK) and $\delta_x = 0.25$ mm. The kinetic energy distribution corresponds approximately to a temperature on the order of 100 nK. This result agrees with the result obtained by estimating mean velocity simply as $v(T) = \omega\delta_x = -3.75$ mm/s (110 nK). However, since the harmonic potential does not act on atoms having large initial velocities, the percentage of the atoms having kinetic energy of 200 nK or more becomes larger than that of the distribution in a thermal equilibrium considered at 100 nK.

3 Analysis under the two-dimensional model

Under the two-dimensional model, it is convenient to perform analysis in a cylindrical coordinate system. Consider that the laser power density distribution is expressed by

$$I(r) = \exp\left[-\frac{r^2}{(\Delta r)^2}\right]$$

$$r^2 = x^2 + y^2 \tag{8}$$

where Δr is a parameter representing the spot size of the laser beam. For centrifugal force, equation (3) used in the one-dimensional model needs to be modified as follows.

$$\frac{d^2 r}{dt^2} = -\omega^2 r \exp\left[-\frac{r^2}{(\Delta r)^2}\right] + \frac{L^2}{Mr^3}$$

$$\omega^2 = \frac{\alpha E_0^2}{M(\Delta r)^2}$$

$$L = xv_y - yv_x \tag{9}$$

Since the dipole force of the laser light is symmetrical to the z-axis, L does not vary. Therefore, the kinetic energy is expressed by

$$K(t) = \frac{M}{2}\left[v_r(t)^2 + \left(\frac{L}{r(t)}\right)^2\right]$$

$$v_r = \frac{xv_x + yv_y}{r} \tag{10}$$

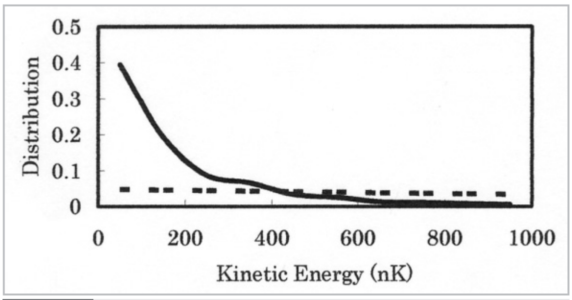

Fig.4 The solid line shows the kinetic energy distribution after irradiation by the collimation laser light under the conditions of equations (11) and (12) (two dimensional). The dotted line shows the kinetic energy distribution in the initial state (in thermal equilibrium at 2.6 μK).

Fig.4 shows the distribution of $K(T)$ when irradiated by the laser light under the following conditions.

$$\Delta r = 1.5 \text{ mm}$$

$$T = 0.1 \text{ s } (\omega = 2\pi \times 2.5 \text{ radian/s}) \tag{11}$$

It is assumed that initial velocity and positional distribution are expressed as follows.

$$\rho_{v_x(0),v_y(0)} = \frac{1}{\pi(\delta v)^2}\exp\left[-\frac{v_x(0)^2 + v_y(0)^2}{(\delta v)^2}\right]$$

$$\delta v = 1.8 \text{ cm/s (equivalent to } 2.6\ \mu\text{K)}$$

$$\rho_{r(0)} = \frac{2r(0)}{(\delta r)^2}\exp\left[-\frac{r(0)^2}{(\delta r)^2}\right]$$

$$\delta r = 0.25 \text{ mm} \tag{12}$$

The distribution of $K(T)$ can be approximately represented by a temperature of 180 nK, which is higher than that calculated by the one-dimensional model. This is because the centrifugal force provides an anharmonic term. However, a temperature lower than that corresponding to the energy of one photon recoil (200 nK) of a Cs atom can still be attained.

4 Considerations

The method used in this paper enables achievement of a temperature below that corresponding to the energy of one photon recoil by decreasing the one-dimensional and two-dimensional velocity components of the atom. Using previously developed VSCPT and Raman cooling, a temperature lower than one photon recoil energy can be obtained. However, each method can only be applied to an atom that has a specific quantum energy structure. The method proposed here is not based on the atomic energy structure, and hence is fundamentally applicable to any atom. Moreover, since only one collimation laser is required, the apparatus is very simple.

In order for the approximation shown by equation (4) to be applicable, the potential depth formed by the dipole force must be larger than the initial kinetic energy of the atom. Then, in order to satisfy the conditions specified by equation (11) using a non-resonant infrared laser, a power of 360 W is required. However, since the collimation effect is dependent on laser frequency (in the case where the detuning is large), a single-frequency laser is not required, and sufficient power can be supplied with a multi-line CO_2 laser. Moreover, since collimation laser light may consist of a standing wave, it is also possible to form a resonator in a collimation component and to raise the power density of that component.

If the laser frequency is brought close to the resonance frequency in a range where the scattering rate becomes sufficiently smaller than $1/T$, collimation can be performed with even lower laser power. The use of 250-mW laser light with detuning of 2 THz satisfies equation (11). At that time, the atoms that undergo scattering within an irradiation time of 0.1 s make up only 8% of the whole.

Although power fluctuations of the collimation laser affect the horizontal velocity component after collimation, if the following equation

$$T\delta\omega << \omega\frac{\delta r}{\delta v} \tag{13}$$

is satisfied, the effect will not be significant. Under the conditions given by equation (12), if the magnitude of fluctuation is sufficiently smaller than 27%, the influence of power fluctuations can be ignored.

5 Conclusions

In this paper, we have proposed and analyzed a novel method for reducing the horizontal velocity component of a launched atomic fountain. From the conditions shown in equation (12), the broadening of the atomic fountain 1 s after the launch becomes 11 mm if the horizontal velocity component is controlled to about 4 mm/s by irradiating a collimation laser on the launched atomic fountain. Assuming that the diameter of the hole of the microwave resonator is 1 cm, 80% of the launched atoms will be able to contribute to the Ramsey signal, and considerable improvement in the S/N ratio can be expected. This collimation method can be used not only in atomic fountains but in on-board satellite space clocks.

Finally, we note that C. Salomon (ENS, France) has devised the same method, independent of the research described herein[9].

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