5 Generation and Dissemination of Time and Frequency Standard

5-1 Algorithm of Ensemble Atomic Time

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This article introduces the algorithm of ensemble atomic time. We can make a more stable time scale than each component atomic clock by calculating the weighted average of each clock. This calculation method is not the only one but the optimum algorithm changes to meet the research purposes and situations. We introduce the ALGOS(BIPM) and AT1(NIST) as the representative algorithms. ALGOS(BIPM) is for TAI (International Atomic Time) which has an excellent long-term stability, and AT1(NIST) is for AT1 time-scale at NIST (National Institute of Standards and Technology) which is a real-time time scale. We also describe about the algorithm of UTC (CRL) and some improvements of it.

Keywords
UTC(Coordinated Universal Time), Time scale algorithm, Ensemble atomic time, UTC(CRL)

1 Introduction

One method of defining time is by counting periodic events (events theoretically regarded to occur at regular intervals). Atomic time is defined by measuring the periods of the electromagnetic wave absorbed or emitted by a specific atom under given conditions. Atomic time, used to define the second, is the most stable time scale currently available, and forms the basis of global standard time[1].

The method or process by which atomic time is defined is referred to as an algorithm for the determination of atomic time. As one second is strictly defined by the transition frequency of a cesium atom, it may seem that atomic time is uniquely determined, leaving no choice as to the selection of method. However, this is not the case. Actual atomic clocks fluctuate from the ideal theoretical state, and this fluctuation influences the output signal. Therefore, a stable time scale must be constructed based on this real-world attribute of atomic clocks.

If you prepare two commercial cesium atomic clocks and compare the output signals, the lengths of the second will not agree perfectly. This is because magnetic environments and electronic circuits differ from one atomic clock to another and because commercial atomic clocks are incapable of self-calibration. In this case, which clock should we trust?

One cannot determine the correct time using these two clocks alone. Yet there is one type of atomic clock that can determine its own error: a primary frequency standard capable of self-calibration[2][3]. Other atomic clocks find their error values by comparing their times with those of primary frequency standards or other standard times.

Yet even if an atomic clock is compared regularly against a primary frequency standard or another standard time value, how do we maintain time between comparisons? We can compare two or more clocks, and select the value of the one showing the least fluctuation (the "master clock" method), or if we have two or more atomic clocks with similar performance, we can average their output signals to obtain a time scale featuring less fluctuation and higher reliability than any individual clock. This method is known as the "ensemble clock" method and is particularly effective with a large number of clocks. A time scale prepared in this way is known as an "average time scale."

A primary frequency standard must occasionally stop regular operation to perform self-evaluation and is therefore unsuitable for maintaining a continuous time. Such a device is instead generally used to calibrate average time scales.

In the calculation of an average time scale, several points must be taken into consideration. For example, the contributions of the component clocks are weighted according to performance, as an unstable clock will impair overall stability. How is this weighting accomplished? How do you evaluate the performance of the clocks? How do you suppress the disturbance caused by the removal of one or more component clocks due to malfunction? The ideal calculation method applied to handle these problems will vary according to the characteristics of the component atomic clocks and the purpose of the time scale. In other words, the ideal algorithm for the determination of atomic time will change according to the situation.

In Chapter **2**, we explain the basic calculations of the ensemble clock method. In Chapter **3**, we describe two typical algorithms: the ALGOS(BIPM) algorithm, which determines international atomic time (TAI) through a post process, and the AT1(NIST) algorithm, a time scale determined in real-time. In Chapter **4**, we introduce the algorithm for the UTC(CRL) generated by the Communications Research Laboratory (CRL) and describe present problems and the methods of improvement. We refer to references[4][6] throughout this report, to references[4][5] in Sections **3.1** and **3.2**, and to references[6][7] in Section **4.1**, but will otherwise omit individual citations.

2 Calculation method of average time scale

2.1 Basic definition

Hereinafter, time is discussed in reference to "ideal time," referring to a perfectly steady time scale, and to the simple term "time," which refers to actual time, which is offset from ideal time. Ideal time is purely conceptual and cannot be obtained in actual calculations or measurement. Below, h_i is the time of clock i, and TA is the average time scale.

The average time scale is theoretically defined as follows (Fig.1 (a)):

$$TA0(t) = \sum_{i=1}^{N} w_i(t)h_i(t), \qquad \sum_{i=1}^{N} w_i(t) = 1 \tag{1}$$

Here, i is the index that identifies each clock, and w_i is the weighting of clock i. When each clock is independent, the weighted average (with optimum weighting) gives a more stable time scale than any of the component clocks alone.

In Equation (1), if clock 1 is removed at point t_0, time $h_1(t_0)$ falls out of the calculation entirely, causing significant time offset in the summation result. What is to be done in this case? The purpose of Equation (1) is initially to reduce fluctuation. Therefore, it must be sufficient to extract the fluctuations and average them alone. Based on this premise, the average time scale may be calculated with the following expression:

$$TA(t) = \sum_{i=1}^{N} w_i(t)\{h_i(t)-h'_i(t)\}, \qquad \sum_{i=1}^{N} w_i(t) = 1 \tag{2}$$

In other words, subtract the predictable variation $h'_i(t)$ of clock i from the actual time $h_i(t)$ of the same clock, treat the difference as the fluctuation, and average all fluctuations, with weighting. This procedure yields the average time scale TA (Fig.1 (b)). The weight $w_i(t)$ and the predictable variation $h'_i(t)$ are determined according to various models. (See Section **2.2**.)

We cannot know the absolute value of $h_i(t)$

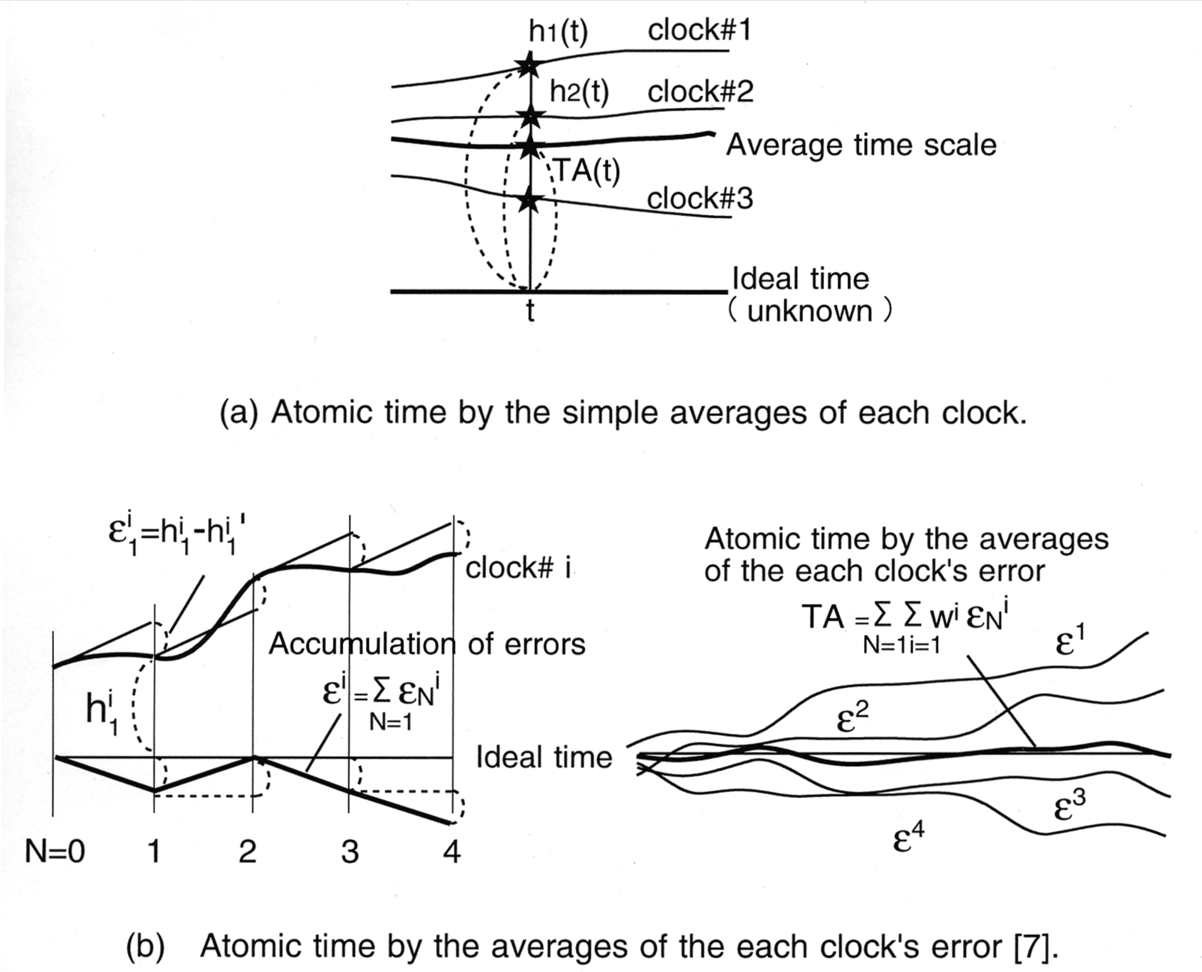

Fig. 1 *Calculation method of average time scale*

because the ideal reference time is unknowable. In other words, we cannot calculate an absolute value for $TA(t)$ from Equation (2). What we can calculate is the time difference x_i between clock i and the average time scale:

$$x_i(t) = TA(t) - h_i(t) \tag{3}$$

x_i can be calculated from the time difference X_{ij} between clocks i and j. X_{ij} is the only value that can be measured and is used as data in the TA calculation:

$$X_{ij}(t) = x_i(t) - x_j(t), \quad i = 1,,,N, \quad i \neq j \tag{4}$$

Equations (2), (3), and (4) yield the following simultaneous equations, which uniquely determine $x_i(t)$:

$$\begin{cases} \sum_{i=1}^{N} w_i(t)x_i(t) = \sum_{i=1}^{N} w_i(t)h'_i(t) & (5) \\ X_{ij}(t) = x_i(t) - x_j(t), \quad i=1,,,N, \quad i \neq j & (4) \end{cases}$$

Equation (4) gives N-1 independent relations containing N clocks; thus, with Equation (5), we have N equations. The unknown quantities are $x_i(t)$ for i = 1 to N, with N the total number of clocks. Therefore, $x_i(t)$ can be determined uniquely from Equations (4) and (5). The explicit expression is as follows:

$$x_j(t) = \sum_{i=1}^{N} w_i(t)\{h'_i(t) - X_{ij}(t)\} \tag{6}$$

Generally, $h'_i(t)$ is predicted with a linear expression:

$$h'_i(t) = x_i(t_0) + y'_i(t)(t - t_0) \tag{7}$$

Here, t_0 is the last period at which x_i is calculated, $x_i(t_0)$ is the time difference between the time given by clock i and TA at t_0, and $y'_i(t)$ is the predicted drift rate (predicted frequency) of clock i. Note that the reference time for $y'_i(t)$ is atomic time itself. That is, the most reliable time scale TA serves as the reference. This means that atomic time is determined in reference to its own past, and if the calculation of $y'_i(t)$ is inadequate, there is a danger of divergence in the above calculation.

To summarize, calculating TA involves calculating the time difference $x_i(t)$ between each clock and the average time scale. The value $x_i(t)$ can be calculated from the time difference $X_{ij}(t)$ between the clocks, the epoch t_0 at which the calculation was conducted, the value $x_i(t_0)$ calculated at t_0, the weight $w_i(t)$ for each clock, and the predicted frequency $y'_i(t)$ for each clock. As the time $h_i(t)$ is unknowable, the numerical value of $TA(t)$ cannot be obtained, but it is possible to calculate the variation in $TA(t)$. In addition, time comparison with the TA of another station can define local TA using their time difference.

2.2 Points to be considered in calculation

Here we must set forth two essential premises in any discussion of an algorithm of ensemble atomic time:

1. The measurement errors of the time difference X_{ij} between the clocks must be negligibly small compared to the noise of the clocks.
2. Each clock must be independent, with no correlation between measured time differences between the clocks.

If these conditions are not satisfied, the method described in Section 2-1 to calculate the average time scale will not be valid.

The ideal algorithm changes according to the type of time scale (standard frequency) required. For instance, is a real-time time scale needed or is an ex post facto report sufficient? What time interval of stability is thought as important? These factors influence the selection of the calculation interval and prediction method for the given frequency. For example, the measurement interval T must be larger than the average time required to suppress errors, and if a real-time time scale is desired, T must be determined in light of this requirement. The frequency prediction method depends on the characteristics of the clocks and the interval of prediction. Even with a single clock, the noise that must be taken into consideration changes with a change in the calculation interval. Following are typical cases:

[1] When white frequency noise is dominant:
* With an averaging time τ of 1 to 10 days, using a commercial cesium clock
* It is appropriate to take the predicted frequency for a certain interval of duration τ as the average of the frequency values for the past intervals of duration τ.

[2] When random walk frequency modulation is dominant:
* With an averaging time τ of 20 to 70 days, using a commercial cesium clock
* It is appropriate to take the predicted frequency for a certain interval of duration τ as being equal to the last frequency value for the immediately prior interval of duration τ.

[3] When linear drift is dominant:
* τ is several days, using a hydrogen maser frequency standard
* It is appropriate to take the predicted frequency for a certain interval of duration τ as the value obtained by subtracting the drift from the frequency value for the immediately prior interval of duration τ.

The weighting of each clock is generally given as the inverse of the frequency variance, and if there are no constraints, the frequency variance of TA is in principle smaller than the variance of each clock. When atomic time itself is the reference time scale, a heavily weighted clock will have a strong influence on atomic time; as a result, the frequency variance of the clock will be underestimated, and its weight will increase even more. It is a common practice to set an upper limit to the weighting value in order to prevent such an imbalance. However, such a constraint could conceivably impair stability. Weighting is assigned in various ways, according to the types and number of clocks. The detection and handling of the abnormal behavior of component ensemble clocks will also serve as an important issue in actual operations.

With these points in mind, next we will discuss two typical algorithms: ALGOS(BIPM) and AT1(NIST).

3 Various algorithms

3.1 ALGOS(BIPM)

ALGOS(BIPM) is a calculation algorithm for the time scale referred to as EAL, which serves as the basis for TAI. The EAL time scale averages a large number of atomic clocks throughout the world (now approximately 250). TAI is obtained by the sum of EAL and a frequency adjustment provided by a primary frequency standard. Taking EAL as $TA(t)$, the definition is as follows:

$$TA(t) = \sum_{i=1}^{N} w_i(t)\{h_i(t) - h'_i(t)\}, \qquad \sum_{i=1}^{N} w_i(t) = 1 \qquad (2)$$

$$x_i(t) = TA(t) - h_i(t) \qquad (3)$$

$$h'_i(t) = x_i(t_0) + y'_i(t)(t - t_0) \qquad (7)$$

and the actual calculation is performed with:

$$X_{ij}(t) = x_i(t) - x_j(t), \quad i = 1,,,N, \quad i \neq j \qquad (4)$$

$$x_j(t) = \sum_{i=1}^{N} w_i(t)\{h'_i(t) - X_{ij}(t)\} \qquad (6)$$

(See Section **2.1**)

The data used are measured every five days. The *TA* calculation is performed every 30 days, and a 30-day interval is used in the calculation. The *TA* value for each five-day period is calculated from the collected values of the seven measured data sets obtained every five days.

$$t = t_0 + mT/6, \quad m = 0,1,,,6, \qquad T = 30\,days \qquad (8)$$

Here, t_0 is the last day of the previous interval, and is 30 days before t, t is the timing for the *TA* calculation, and T is the calculation interval. Within the same interval, the predicted frequency and weighting values are not changed.

The predicted frequency $y'_i(t)$ of the present interval $[t_0, t_0+T]$ is unchanged from the frequency $y'_i(t_0)$ of the previous interval $[t_0-T, t_0]$. This method is adopted because the main noise of the cesium atomic clock is random walk frequency modulation when the calculation interval is 30 days. (See Section **2.2**)

$$y'_i(t) = y_i(t_0), \qquad t = t_0 + mT/6, \quad m = 0,1,,,6 \qquad (9)$$

$y_i(t_0)$ is the least mean square gradient obtained from the 7 points of $x_i(t)$ in the previous interval $[t_0-T, t_0]$. In principle, it would be best to calculate the predicted frequency using $y_i(t_0)=\{x_i(t_0+T)-x_i(t_0)\}/T$ in the case of random walk frequency modulation, but the risk of abnormal prior data is so great that the least mean square method is adopted.

Weighting in the present interval $[t_0, t_0+T]$ is calculated as follows:

[1] Obtain $x_i(t)$ of the present interval using Equations (6) and (7), with the weighting of the previous interval $[t_0-T, t_0]$ and the predicted frequency $y'_i(t_0)$ of Equation (9).

[2] Calculate the frequency $y_i(t_0+T)$ of the present interval from the gradient of the least mean square fitting for the obtained $x_i(t)$.

[3] Calculate the classical frequency variance $\sigma_i^2(12,T)$ for the past year from the frequencies of the current and the past 11 intervals:

$$\sigma_i^2(12,T) = \frac{1}{12}\sum_{k=1}^{12}\{(y_i^k - \langle y_i^k\rangle)^2\} \qquad (10)$$

Here, k is the index that indicates the section of interval, and y_i^k is the frequency of clock i in interval k.

[4] Calculate $w_i(t)$ from $(\sigma_i^2(12,T)$:

$$w_i(t) = p_i / \sum_{i=1}^{N} p_i, \quad p_i = \frac{1}{\sigma_i^2(12,T)}, \quad \sum_{i=1}^{N} w_i(t) = 1 \qquad (11)$$

The ensemble clocks that constitute TAI can be classified into three categories: high-performance HP5071A cesium clocks, hydrogen masers, and others. Although the concentration of weighting on a small number of clocks presents problems, we want to ensure that clocks with higher stability are weighted as heavily as possible. To satisfy this requirement, the definition of weighting has gone through several changes and is now elaborated as follows[8]:

$$w_i(t) \geqq w_{max} \quad \text{for } w_i(t) = w_{max} \qquad (12)$$

$$w_{max} = A/N \qquad (13)$$

Here, N is the number of clocks and A is an empirically determined constant. From Equations (11), (12), and (13), the threshold of $\sigma_i^2(12, T)$ to obtain the largest wmax becomes smaller as the value for A becomes larger, which gives only stable clocks maximum weighting. To differentiate the clocks, a larger A is more desirable, but if it is too large, the number of clocks that can receive maximum weighting is reduced. The balance between these requirements determines the value of A.

To eliminate abnormal data from the ensemble component clocks, the method adopted here gives a weighting value of 0 if the frequency $y_i(t_0+T)$ of the present interval is significantly different from the frequency average $< y_i >_{11}$ of the past 11 intervals:

$$w_i(t)=0 \text{ if } y_i(t_0+T)-<y_i>_{11} \; > \; 3s_i(12,T). \qquad (14)$$

When the noise consists of random walk frequency modulation, the 12-value variance $s_i^2(12, T)$ can be estimated from the variance $\sigma_i^2(11,T)$ of the past 11 samples:

$$s_i2(12,T) = \frac{12}{11}\sigma_i2(11,T) = \frac{1}{11}\sum_{k=1}^{11}\{(y_i^k - \langle y_i^k\rangle)2\} \qquad (15)$$

One of the features of ALGOS is that it is based on the post process and can easily detect abnormal data, compensating for the fact that the constructed time scale (EAL, TAI) has no real-time characteristics. Since clock data throughout the world is mainly compared using the GPS common-view method, the interval was set at 30 days, the number of days required to average GPS common-view measurement data. As the accuracy of time comparison increases in the future, the calculation interval will be reduced. Because the variance $\sigma_i^2(12,T)$ used to determine weighting is calculated from all data for the previous year, ALGOS offers an advantage in that it is not easily influenced by seasonal fluctuation.

3.2 AT1(NIST)

AT1(NIST) is a real-time time scale that consists of approximately 10 commercial cesium clocks. The definition is given by the following equations:

$$TA(t) = \sum_{i=1}^{N} w_i(t)\{h_i(t)-h'_i(t)\}, \qquad \sum_{i=1}^{N} w_i(t) = 1 \qquad (2)$$

$$x_i(t) = TA(t) - h_i(t) \qquad (3)$$

$$h'_i(t) = x_i(t_0)+y'_i(t)(t - t_0) \qquad (7)$$

and the actual calculation is performed according to

$$X_{ij}(t) = x_i(t) - x_j(t), \quad i = 1,..,N, \quad i \neq j \qquad (4)$$

$$x_j(t) = \sum_{i=1}^{N} w_i(t)\{h'_i(t) - X_{ij}(t)\} \qquad (6)$$

(See Section **2.1**)

The interval for measurement is 2 hours, and the interval for TA calculation is also 2 hours. Accordingly, the interval of calculation is also 2 hours. One item of measurement data is used in the calculation, so the atomic time of the present interval is calculated only from the data relating to the immediately prior interval:

$$t = t_0+mT, \quad m=1, \qquad T=2hours \qquad (16)$$

The last calculation time t_0 of the previous interval is 2 hours earlier, t is the timing of the TA calculation, and T is the calculation interval. Weighting and predicted frequency are updated at each interval (every 2 hours).

The predicted frequency $y_i'(t)$ is calculated by the average of the past and the present values, with exponential weighting. $y_i'(t)$ is used to calculate the following $y_i'(t+T)$:

$$y'_i(t) = \frac{1}{m_i+1}(y_i(t) + m_i y'_i(t_0))$$

$$y_i(t) = \frac{x_i(t) - x_i(t_0)}{t - t_0} \qquad (17)$$

When the main noise is either white and random walk frequency modulation, m_i is obtained by the following equation:

$$m_i = \frac{1}{2}\left[-1+\left\{\frac{1}{3}+\frac{4}{3}\frac{\tau_{min,i}2}{T2}\right\}^{1/2}\right] \qquad (18)$$

$\tau_{\min,i}$ is the period in which each clock is the most stable.

Weighting at time t is calculated from the values in the previous interval:

$$w_j(t) = p_i/\sum_{i=1}^{N} p_i, \quad p_i = \frac{1}{\langle \varepsilon_i 2 \rangle}, \quad \sum_{i=1}^{N} w_i(t) = 1 \qquad (19)$$

$$\langle \varepsilon_i 2 \rangle_t = \frac{1}{N_\tau + 1} \left(\varepsilon_i 2 + N_\tau \langle \varepsilon_i 2 \rangle_{t0} \right) \qquad (20)$$

$$|\varepsilon_i| = |h'_i(t) - x_i(t)| + K_i \qquad (21)$$

$$K_i = 0.8\, p_i \langle \varepsilon_i 2 \rangle^{1/2} \qquad (22)$$

The constant N_τ is set between 20 and 30 days. This reduces the influence of the past value. ε_i is the difference between the predicted and estimated time. K_i is the correction term added considering the correlation between clock i and TA. K_i is negligible when the number of clocks is large but this term is necessary when the number of clocks is small (up to approximately 10 clocks).

AT1(NIST) does not record the absolute values of the past frequencies; it looks only for frequency variations. This method is similar to that seen in the Allan variance. However, with this method, there is a possibility of losing the long-term fluctuation information. As it does not perform any recalculation or additional calculations, the algorithm features excellent real-time characteristics. The adoption of an exponential filter is effective in reducing the number of accidents, but it cannot eliminate long-term fluctuations such as seasonal fluctuation. To eliminate abnormal data, this algorithm detects frequency steps using a threshold equivalent to four times the frequency noise (although this threshold is not included in the above equations).

4 UTC(CRL)

In this chapter, we will introduce UTC(CRL), the real-time atomic time of the CRL. The atomic time algorithm with a cesium-clock ensemble was adopted in 1986, and this time scale has been maintained with no change to the calculation method. However, recent improvements in atomic clocks are beginning to require modification to the algorithm. In Section **4.1**, we will describe the present method of calculation, and in Section **4.2**, we will discuss its problems and improvements to be made.

4.1 Present calculation method

UTC(CRL) is a real-time time scale that calculates time using 12 commercial cesium atomic clocks at the CRL Koganei Headquarters. The definition is expressed by the following equations:

$$TA(t) = \sum_{i=1}^{N} w_i(t)\{h_i(t) - h'_i(t)\}, \qquad \sum_{i=1}^{N} w_i(t) = 1 \qquad (2)$$

$$x_i(t) = TA(t) - h_i(t) \qquad (3)$$

$$h'_i(t) = x_i(t_0) + y'_i(t)(t - t_0) \qquad (7)$$

and it is calculated using the following expressions:

$$X_{is}(t) = x_i(t) - x_s(t), \quad i = 1,,,N, \quad i \neq j, \qquad (4)$$

$$x_s(t) = \sum_{i=1}^{N} w_i(t)\{h'_i(t) - X_{is}(t)\}, \qquad (6)$$

$$x_i(t) = x_s(t) + X_{is}(t). \qquad (23)$$

(See Section **2.1**) The relationships between the values are indicated in Fig.2.

The interval for measurement is 1 day, and the TA calculation interval is also 1 day. (In fact, time differences between the clocks are measured every 4 hours, but only the data at 0 h UTC is used in the calculation.) Thus let us take 1 day as the interval. The only data used in the calculation is the single set of data for the previous day.

$$t = t_0 + mT, \quad m = 1, \qquad T = 1\,days \qquad (24)$$

The last calculated time t_0 for the previous interval is 1 day earlier, t is the timing of the TA calculation, and T is the calculation interval. Weighting and predicted frequency are updated every interval (every day).

The predicted frequency $y'_i(t)$ after 61 days from the beginning of the calculation is calculated from the following definition:

$$\begin{cases} y'_i(t) = y'_i(t-T) \; if \; |\, y10'_i(t) - y'_i(t-T) \,| \leq 1\,x10^{-12}, \\ y'_i(t) = y10'_i(t)/\{1 - w_i(t)\} \; if \; |\, y10'_i(t) - y'_i(t-T) \,| > 1x10^{-12}. \end{cases} \qquad (25)$$

Here, $y10'_i(t)$ is the rate for the 10 previous days.

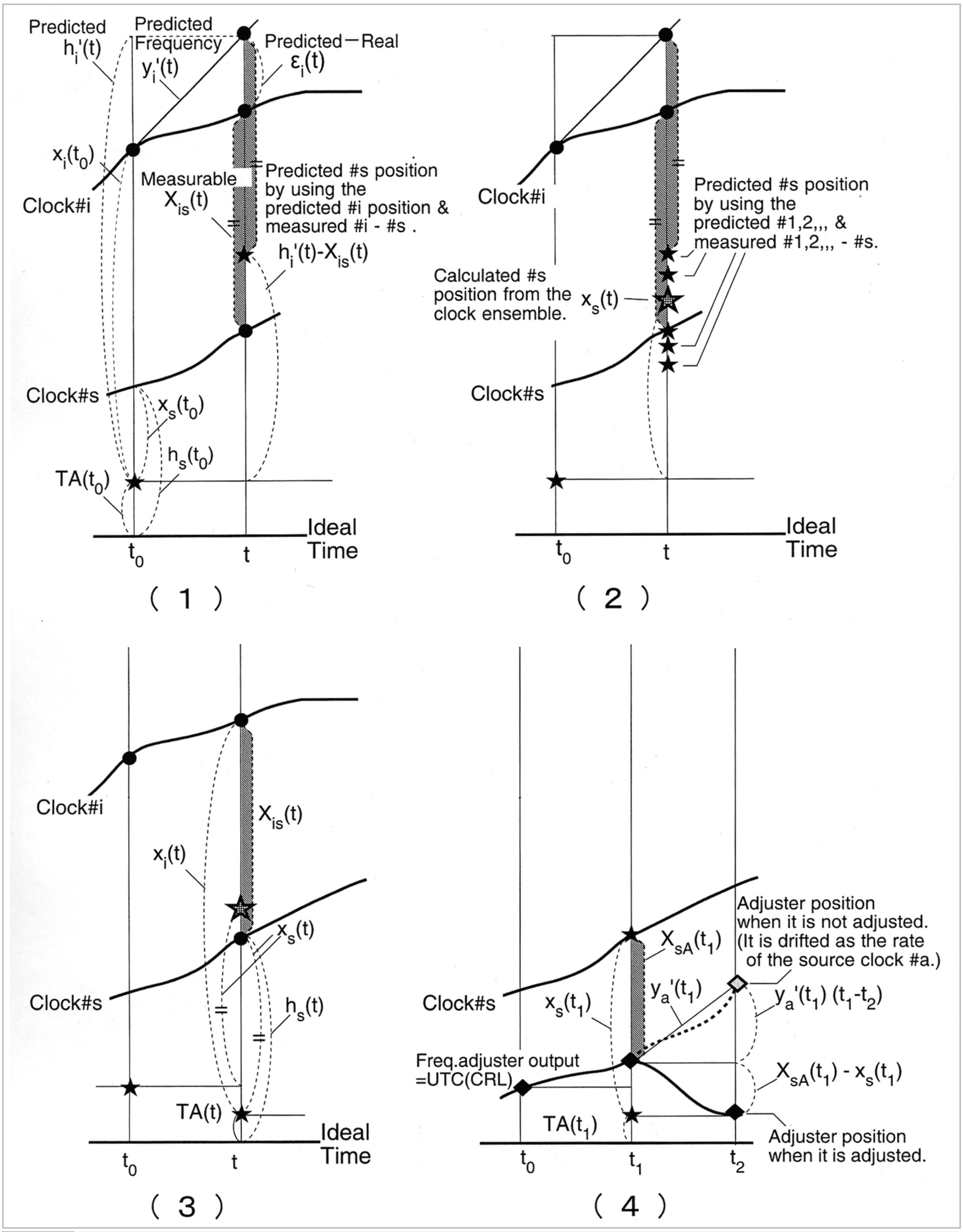

Fig.2 *Relationships between the values relating to calculation of UTC(CRL)*

$$y10'_i(t) = \{x_i(t) - x_i(t - 10T)\} / 10T \qquad (26)$$

The initial value for $y'_i(t)$ should be the rate calculated from the data for the first 60 days. In this equation, unless the rate for the 10 previous days deviates from the rate for the previous day by 1×10^{-12}, $y'_i(t)$ continues to take the same value as that of the previous day. This offers an advantage in that the rate cannot easily be influenced by abnormal fluctuation of the clocks, but there is also a disadvantage: the latest fluctuations are not readily reflected in the calculation. With recent improvements in the performance of atomic clocks, this method is no longer ideal. We will discuss modification to the method in Section **4.2**.

When determining weighting values, unbiased clock variance[6] [7] is introduced in order to avoid excess concentration on a specific clock:

$$w_j(t) = p_j / \sum_{i=1}^{N} p_i, \quad p_i = \frac{1}{z_i}, \quad \sum_{i=1}^{N} w_i(t) = 1 \tag{27}$$

$$\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_4 \end{pmatrix} = \begin{pmatrix} (1-w_1(t_0))^2 & (w_2(t_0))^2 & \dots & (w_N(t_0))^2 \\ (w_1(t_0))^2 & (1-w_2(t_0))^2 & \dots & (w_N(t_0))^2 \\ \vdots & \vdots & \vdots & \vdots \\ (w_1(t_0))^2 & (w_2(t_0))^2 & \dots & (1-w_N(t_0))^2 \end{pmatrix}^{-1} \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_4 \end{pmatrix} \tag{28}$$

where p_i is the Allan variance of $x_i(t)$ at τ =10 days, $w_i(t_0)$ in the matrix is the weighting for the previous day, and z_i is the unbiased clock variance.

From Equation (3), atomic time $TA(t)$ is obtained as an actual signal by correcting the output of clock $h_i(t)$ by the calculated value $x_i(t)$. However, we cannot artificially adjust the frequency of the cesium atomic clocks, as these clocks serve as the basis for the ensemble calculation. Thus, we modify the output of the frequency adjuster corresponding to a cesium clock and regard this adjusted output as the signal $TA(t)$. Maintaining this signal to trace UTC, we regard this signal as the actual signal of UTC(CRL). Denoting the output of the frequency adjuster as $h_A(t)$ [as noted, this value represents UTC(CRL)] and the time difference relative to the reference clock s as $X_{sA}(t)$, $TA(t)$ is expressed as:

$$TA(t) = h_A(t) + x_A(t) = h_A(t) + \{x_s(t) - X_{sA}(t)\} \tag{29}$$

Because TA is calculated and the frequency is adjusted once each day, a correction value is given to maintain the present value for $h_A(t)$ until the following day. The frequency adjuster drifts in accordance with the rate of its referred oscillator, cesium clock a; thus, taking the adjustment value $y_{adj}(t)$ into consideration as well, the output of the frequency adjuster for the next day is expressed as follows:

$$h_A(t+T) = h_A(t) + y'_a(t)T + y_{adj}T \tag{30}$$

Here, $y'_a(t)$ is the rate of clock a, and is calculated from the 10 previous days. The value for $y_{adj}(t)$ is determined such that the value of $h_A(t+T)$ in Equation (30) equals the value of $TA(t)$ in Equation (29):

$$y_{adj}(t) = \frac{X_{sA}(t) - x_s(t)}{T} - y'_a(t) \tag{31}$$

In actual operation, we adjust the frequency whenever necessary using the UTC-UTC(CRL) time-difference value included in the Circular-T report published monthly by the BIPM, in order to minimize time discrepancies relative to the UTC.

Fig.3 shows the UTC-UTC(CRL) fluctuation in 2001 and 2002. The details of concrete generating system of UTC(CRL) are discussed in article 5.2 of this special issue.

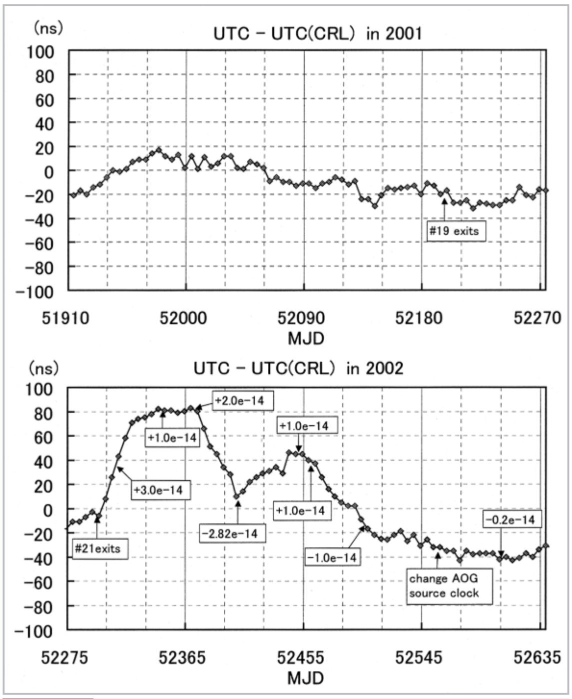

Fig.3 *Time difference between UTC(CRL) and UTC(Circular T reported values)*

4.2 Improvements to UTC (CRL)

One of the problems with the present UTC(CRL) is that the atomic time drifts significantly when a clock is removed from the ensemble. For example, in 2002, there was a drift due to the failure of Clock 21 (Fig.3). This drift occurs because the rate calculation method is no longer applicable in practice. Although the atomic time is calculated by:

$$TA(t) = \sum_{i=1}^{N} w_i(t)\{h_i(t)-h'_i(t)\}, \qquad \sum_{i=1}^{N} w_i(t) = 1 \qquad (2)$$

$$h'_i(t) = x_i(t_0)+y'_i(t)(t - t_0) \qquad (7)$$

if $y'_i(t)$ does not adequately express the rate of each clock, the error $h_i(t)-h'_i(t)$ corresponding to the actual time difference between the clocks and the predicted value increases. When such a clock with a significant error is removed from the ensemble calculation, a significant effect is produced in *TA*. In the present method of rate calculation, any fluctuation equal to or less than 10^{-12} is not reflected in the calculation; thus frequency errors of a magnitude close to 10^{-12} may accumulate daily in the worst case (see Section **4.1**).

In the past, when clock performance was lower than it is today, the present method (which is not easily influenced by the fluctuation of individual clocks) was effective in minimizing the risk of error. However, the performance and reliability of atomic clocks have recently improved, and it therefore appears advisable to employ a calculation method that more accurately reflects clock fluctuation.

For trial operation, we prepared an RTA30 test time scale using the rate for the past 30 days $y30'_i(t) = \{x_i(t)-x_i(t-30T)\}/30T$ instead of the present $y'_i(t)$, and examined the effect of clock removal. Fig.4 shows the results of the simulation. The drift caused by the clock removal is clearly less with RTA30 than with UTC(CRL). Similar tests were conducted using rates for the past 10 days and for the past 60 days, but the results for the past 30 days are the best. This is probably because the frequency stability of a cesium clock is greatest at approximately 30 days.

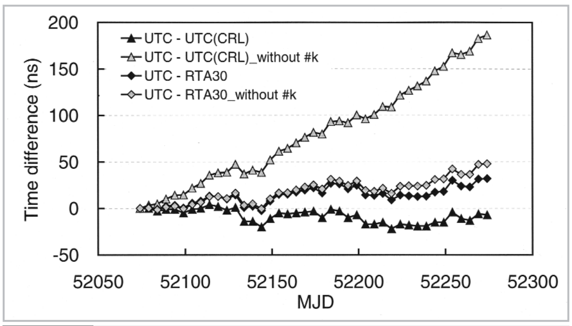

Fig.4 *Comparison of the effects of clock removal on UTC(CRL) and on the test time scale*

Another problem is seen in the short-term stability of UTC(CRL). UTC(CRL) consists of the output of the frequency adjuster, and the adjustment value is calculated daily using Equation (31). Fig.5 shows the difference between the frequency adjustment value and the value for the previous day. Fluctuations exceeding 1×10^{-13} sometimes appear. As the frequency stability of a cesium clock is approximately 3×10^{-14} for a one day interval, we want to maintain the daily frequency adjustment value below this threshold.

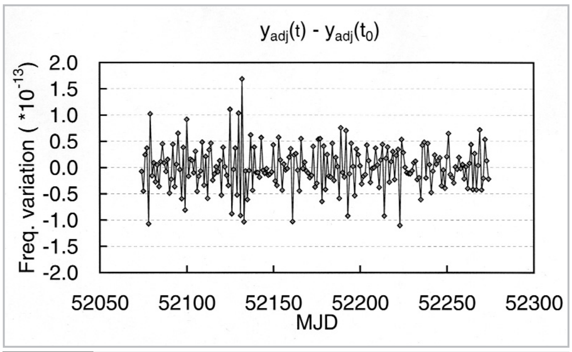

Fig.5 *Fluctuation of AOG frequency adjustment with UTC(CRL)*

According to Equation (31), the frequency adjustment value y_{adj} is determined as the sum of three components: the measured time difference X_{sA} between the reference clock s and the frequency adjuster output [UTC(CRL)], the calculated time difference x_{sA}, and the rate $y'_a(t)$ of the referred oscillator clock of the frequency adjuster. We separated these three components and examined their individual fluctuations. The results show that the main cause of the dispersion was the measured time difference X_{sA}. As we employed a single measurement of 1 pps for X_{sA} without averaging, it is probable that measurement errors and short-term instability are directly reflected in the calculation. However, replacing the measurement instrument or the measurement method has a significant effect on the entire system. As an alternative method, we calculated a daily representative data point by least mean square fitting of all measured data (total of 6 data points with measurement every 4 hours) and applied this value in the place of the present value, but the results show little improvement. Six data points are probably insufficient. It is not appropriate to increase the number of days in least mean square fitting due to the nature of the noise involved; nor have we been able to improve results through modification of the simulation.

When we observe Fig.5 closely, we see that y_{adj} has a tendency to alternate between positive and negative values from day to day. This indicates that the present y_{adj} is adjusted excessively. When we decrease the frequency adjustment value to half of the present value, the frequency fluctuation is suppressed (Fig.6). As the time difference from UTC remains approximately the same as with the present adjustment value, we have concluded that this method is effective in practice.

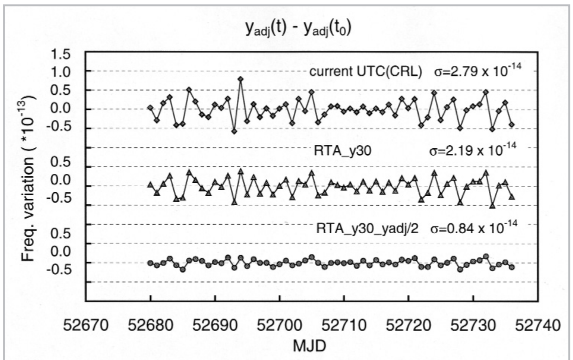

Fig.6 *Fluctuation of AOG frequency adjustment values for UTC(CRL) and test time scales*

The short- and medium-term stability of the time scale with these two improvements (modification of rate calculation method and revision of the frequency adjustment value) is examined. The frequency stability relative to the hydrogen maser is calculated for the three time scales: present UTC(CRL), the RTA_y30 time scale (with improved rate calculation method only), and the RTA_y30_adj/2 time scale with 1/2 y_{adj} added to RTA_y30 (Fig.7). The stability of RTA_y30 is better than the present UTC(CRL) for all intervals from Day 1 to Day 8. This is probably due to the reduction in the predicted error resulting from the modification of the rate calculation method. The daily frequency stability shows improvement for RTA_y30_adj/2 compared to RTA_y30, but no difference in results is seen after 1 day. As the change in the adjustment value y_{adj} should improve only daily instability, with no effect on other intervals, this result is appropriate.

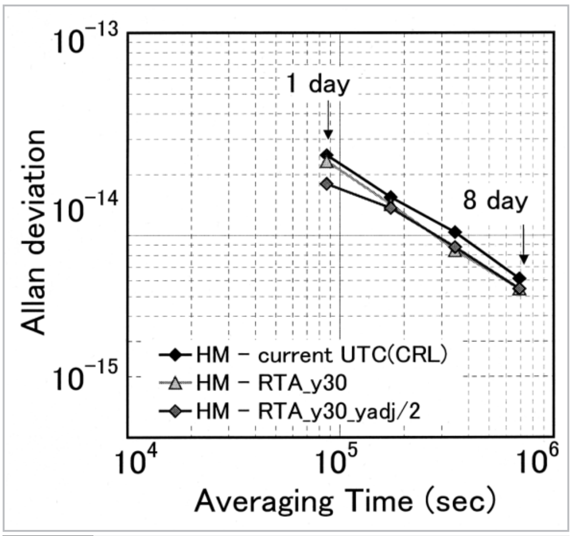

Fig.7 *Comparison of frequency stability for UTC(CRL) and test time scales*

The time scale with these two improvements has been in operation for the stand-by system since April 2003, and will soon be applied to the active system [9].

5 Summary

The ensemble clock method can produce a virtual clock with better stability than available even with the most stable single clock. Under the two typical algorithms, ALGOS(BIPM) and AT1(NIST), the basic premises for the construction of a time scale, are the same. That is, the average time scale is calculated by averaging two or more weighted atomic clocks. In the course of the calculation, the drift rate of each clock relative to atomic time is eliminated beforehand, and the remaining fluctuation is subject to weighted averaging. The reference time is the average time scale itself, which is the most stable one, and the process is necessary in which the calculation is performed in reference to the average time scale itself. Thus, this method presents risks of divergence and imbalance; appropriate care should be taken. This type of calculation method has also been adopted by other standardization organizations. Nevertheless, algorithms other than those we have described are available, such as the algorithm in which the weighting calculation is different for short and long periods, or the algorithm that employs the Kalman filter. Such algorithms are not addressed in this report.

The optimal calculation method in practice will vary depending on the type and the number of atomic clocks, on whether or not a real-time component is necessary, and on which time span requires the greatest stability. Different organizations have applied different methods with respect to the measurement interval for time difference data, the time interval for calculating atomic time, the length of the rate calculation interval, the method of calculation of this interval, and the weighting method. For example, the real-time UTC (CRL) time scale adopts a calculation method similar to that of AT1 (NIST), but with an original method of weighting calculation. Further, for a time scale providing standard time, reliability is as important as quality. For example, the elimination or suppression of the effects of accidents are significant issues. Depending on the purpose, a method featuring lower risk of error at the time of accidents may be a wiser choice, even if some quality might be sacrificed. There is no single correct answer in the determination of the most appropriate algorithm; each method must be selected based on the applicable conditions or purposes.

We have maintained stable operation of UTC(CRL) for 20 years without major modifications to the established calculation algorithm. However, in the long course of operations, several problems have become clear. For this report, we have explored the causes of these problems, proposed methods for improvements, and tested their effectiveness. These improvements will soon be implemented, in conjunction with the preparation of a new system to generate UTC(CRL). Development of this system will involve the introduction of a hydrogen maser to improve short-term stability; this will require modification of the present algorithm. Various simulations are currently underway, as part of our efforts to advance the required research and to ensure the soonest practical application of the revised system.

References

1 T. Morikawa, "Definitions of Time and Frequency Standard", This Special Issue of CRL Journal.

2 K. Fukuda et al., "Optically Pumped Cesium Primary Frequency Standard", This Special Issue of CRL Journal.

3 M. Kumagai et al., "Development of Atomic Fountain Primary Frequency Standard at CRL", This Special Issue of CRL Journal.

4 C. Thomas, P. Wolf, and P. Tavella, "Time scale", BIPM Monographie 94/1, pp.23-32, 1994.

5 P. Tavella and C. Thomas, "Comparative study of time scale algorithms", Metrologia, Vol. 28, pp.57-63,1991.

6 K. Yoshimura et al., "SHYUUHASUU TO JIKAN",The Institute of Electronics, Information and Communication Engineers, 1989. (in Japanese)

7 K. Yoshimura, "GENSHIJI NO ALGORITHM", Review of the RRL, Vol.29, No.149, pp.175-192, 1983. (in Japanese)

8 J.Azoubib, "A Revised Way of Fixing an Upper Limit to Clock Weights in TAI Computation", Proc. of 32nd PTTI meeting, pp.195-209, 2000.

9 Y. Hanado, K. Imamura, and M. Imae, "Upgrading of UTC (CRL)", Proc. of FCS2003, printing.

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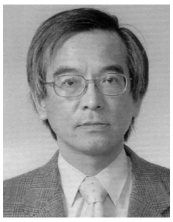
Millisecond Pulsar Timing Measurements, Algorithm of Ensemble Atomic Time



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