

2-2 Power Meter Calibration

2-2-1 Power Meter Calibration 1 (1 mW, 50 ohm)

Tsutomu SUGIYAMA, Kojiro SAKAI, Kouichi SEBATA, Iwao NISHIYAMA, and Katsumi FUJII

NICT performs calibration services for power meters with coaxial sensors in the frequency range from 100 kHz to 50 GHz, and for those with waveguide sensors from 50 GHz up to 170 GHz. In order to ensure the accuracy, the simultaneous comparison and substitution method is adopted as the principle of calibration. The calibration system has the expanded uncertainty of 0.66 % (coverage factor $k=2$) at 100 MHz. In this report, we describe the calibration methods, systems, and uncertainties in the frequency range 100 kHz to 110 GHz.

1 Introduction

The output levels and modulation methods are established for TV and radio broadcast waves, and various radio waves produced from wireless devices such as mobile phones and radar. For measurement of their output levels, high-frequency power meters (hereinafter, “Power Meters”) are mainly used. Power Meters are also used when calibrating measurement instruments, such as signal generators and spectrum analyzers. Power Meters are important measuring instruments used in evaluating characteristics of wireless devices, and in calibration of various measuring instruments, so their calibration is required in order to measure accurately.

As of November 2016, NICT is calibrating Power Meters in frequencies from 100 kHz to 170 GHz. For sensors (Type-N 50 Ω , 3.5 mm, 2.4 mm) with various coaxial connectors as input terminals (hereinafter abbreviated as “Coaxial Sensors”), NICT is calibrating from 100 kHz to 50 GHz. For sensors with waveguides as input terminals (V-Band (50–75 GHz), W-Band (75–110 GHz), D-Band

(110–170 GHz)) (hereinafter abbreviated as “Waveguide Sensors”), NICT is calibrating up to 170 GHz.

This report describes calibration methods for Power Meters from 100 kHz to 110 GHz frequencies (1 mW or less, Coaxial Sensors and Waveguide Sensors), and calibration systems and uncertainty calculation methods.

2 Definitions

General Power Meters have a structure divided into the indicator part and sensor. Both are connected by a cable (Fig. 1). Equation (1) shows the relationship between the instruction value P_M and incident power P_{in} into the sensor.

$$K = \frac{P_M}{P_{in}} \quad (1)$$

Here, K is the calibration factor, and calibration is seeking the value of this K . General Power Meters contain a reference signal source (50 MHz, 1 mW), and when using a Power Meter, a sensor is first attached to the reference signal source, and the 1 mW value is aligned. Therefore, the calibration factor also includes the precision of this reference signal source, and at NICT, the indicator part and sensor are calibrated as one unit.

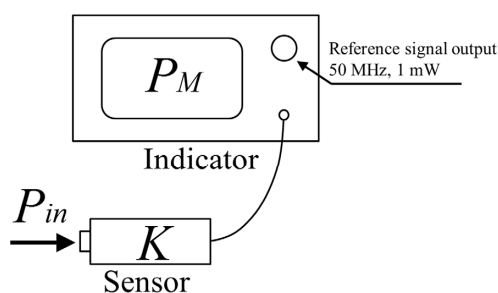


Fig. 1 Basic components of high-frequency Power Meters

3 Calibration methods

3.1 Simultaneous comparison and substitution method

There are various Power Meter calibration methods: comparison method, simultaneous comparison method, simultaneous comparison and substitution method, etc. The simultaneous comparison and substitution method has

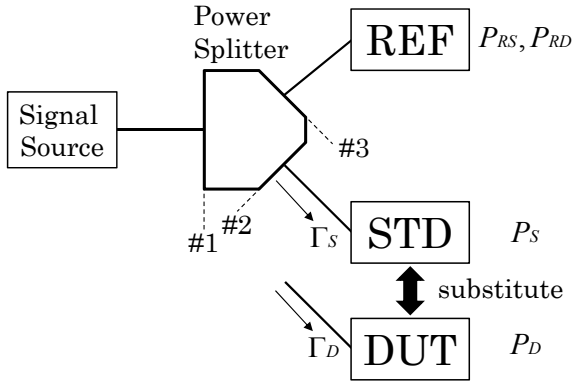


Fig. 2 Simultaneous comparison and substitution method

advantages: it is not affected by reflection of signal source, is strong against output variation of the signal source, etc. NICT uses the simultaneous comparison and substitution method for calibration of Power Meters (1 mW or less, Coaxial Sensors and Waveguide Sensors).

Figure 2 shows a conceptual diagram of the simultaneous comparison and substitution method. The signal from the signal source is connected to Port #1 of the power splitter, and power that goes through the power splitter is distributed into Test Port #2 and Reference Port #3.

After that, calibration is done by the steps below.

- 1) Connect the reference device (REF) to Reference Port #3 (do not remove until calibration ends)
- 2) Connect the standard device (STD) to Test Port #2
- 3) Adjust the signal source connected to Port #1 so the STD measurement value of Test Port #2 is 1 mW.
- 4) Seek the ratio $R_S (= P_{MS} / P_{RS})$ of STD measurement value P_{MS} vs. REF measurement value P_{RS} at that time
- 5) Change STD connected to Test Port #2 to Device Under Test (DUT)
- 6) Make the signal source the same output as 3)
- 7) Seek the ratio $R_D (= P_{MD} / P_{RD})$ of DUT measurement value P_{MD} vs. REF measurement value P_{RD} at that time
- 8) From 4) and 7), use Equation (2) to determine the calibration factor K_D of DUT. [1]

$$K_D = K_S \frac{R_D}{R_S} M \tag{2}$$

where, K_S is the calibration factor of STD used by an upper-level calibration organization.

$$R_S = \frac{P_{MS}}{P_{RS}} \tag{3}$$

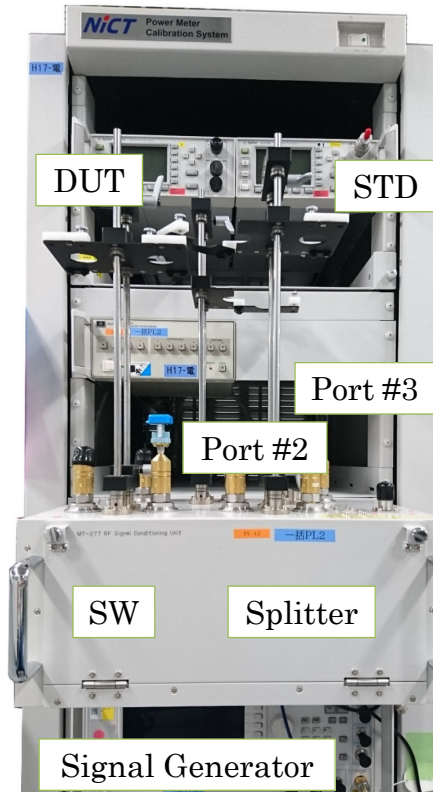


Fig. 3 Calibration system

$$R_D = \frac{P_{MD}}{P_{RD}} \tag{4}$$

Here, mismatch M is expressed in the following equation from the equivalent signal source reflection coefficient Γ_{g2} , the reflection coefficient Γ_S of STD, and the reflection coefficient Γ_D of DUT.

$$M = \left| \frac{1 - \Gamma_{g2} \Gamma_D}{1 - \Gamma_{g2} \Gamma_S} \right|^2 \tag{5}$$

Γ_{g2} can be obtained from S parameters of the power splitter.

$$\Gamma_{g2} = S_{22} - S_{32} \frac{S_{21}}{S_{31}} \tag{6}$$

3.2 Calibration of power meter using coaxial sensor

For a Coaxial Sensor, it is possible to calibrate Power Meters with 1 mW power, 100 kHz to 50 GHz frequency [1]. Figure 3 shows a calibration system photo, with a block diagram in Fig. 4.

This calibration system uses the simultaneous comparison and substitution method. Output from the signal generator is input via a switch into the power splitter (Port #1), and the power splitter splits it into Reference Port #3 and Test Port #2. The calibration steps are the steps explained in the steps of the simultaneous comparison and

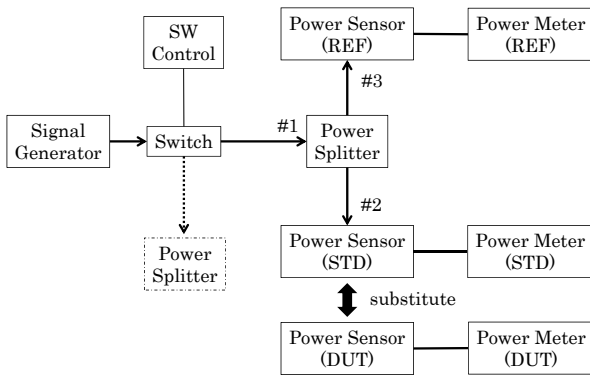


Fig. 4 Calibration system block diagram

substitution method.

One advantage of this calibration system is it is designed with the calibration system's Reference Port #3 and Test Port #2 facing upward, so each port plane and sensor's connector plane contact equally. Moreover, by switching the switches, one can select the power splitter corresponding to the frequency range and the output port (test port or reference port) corresponding to the sensor's connector, and one can thereby calibrate various connectors (Type-N 50 Ω , 3.5 mm, 2.4 mm) from 100 KHz to 50 GHz. When connecting each connector, to ensure reproducibility, we use a torque wrench, and always tighten to the same torque.

3.3 Calibration of power meter using waveguide sensor

In the case of a Waveguide Sensor, calibration of the Power Meter is possible with 1 mW V-Band, and 0.1 mW W-Band[2]. There are different calibration systems for V-Band and for W-Band. Figure 5 shows a photo for the W-Band system, and Fig. 6 shows its block diagram.

This system also uses the simultaneous comparison and substitution method for its calibration method. However, the Coaxial Sensor's calibration system used a power splitter, but this system uses a directional coupler (degree of coupling: 6 dB) instead of a power splitter. Output from the signal generator is multiplied via a multiplier, multiplied 4 times in the case of V-Band, and multiplied 6 times in the case of W-Band, then input into the directional coupler (Port #1). In the directional coupler, the signal is split into each, and in the direction of the traveling waves, two isolators are also connected, and that output terminal is Test Port #2. Also, Reference Port #3 is a terminal in the direction of the reflected wave of the directional coupler.

Like the Coaxial Sensor calibration system, an advantage of this calibration system is that it is designed with the calibration system's Reference Port #3 and Test Port #2

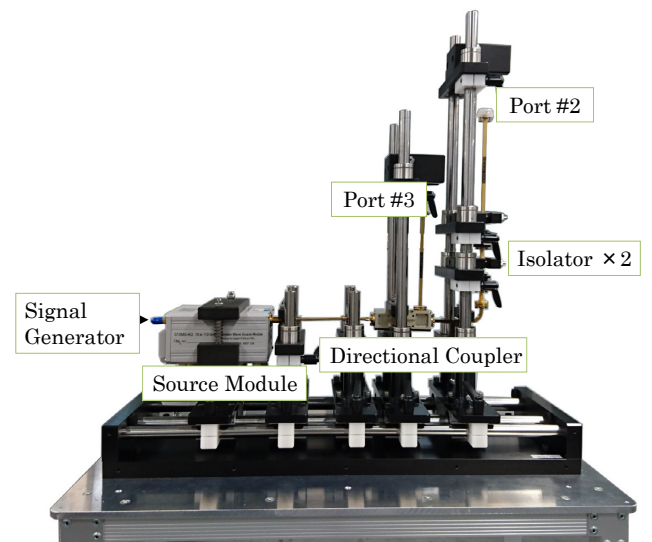


Fig. 5 Calibration system (W-band)

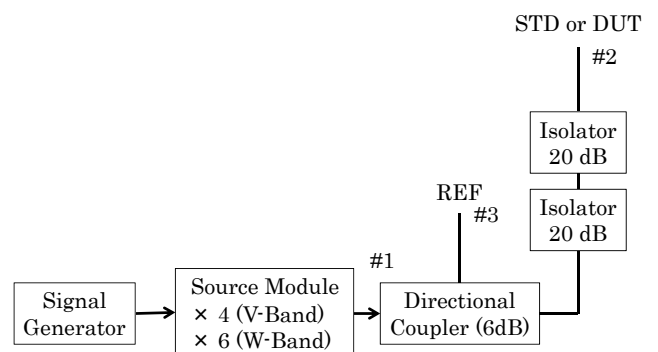


Fig. 6 Calibration system (W-band) block diagram

facing upward, so each port and Waveguide Sensors' waveguides contact equally. Moreover, the calibration system's waveguides are very delicate, so it has fixtures to secure the Waveguide Sensor to prevent shaking.

3.4 Calibration value

Calibration is done by the steps described above. and P_{RD} are each measured 100 times, that average is obtained, and Equation (2) is used to calculate the calibration value. The calibration factor is calculated with M as 1, and the effect due to considering $P_{RS} M$ as 1 is evaluated as uncertainty.

However, at high frequencies, we can no longer ignore M 's effect on calibration value, so Γ_{g2} , Γ_D and Γ_S (all of them complex quantities) are used to calculate the value of M defined in Equation (5), that result is applied in Equation (2), and the corrected calibration factor is calculated. Figure 7 is a graph of values of M when DUT (2.4 mm connector, 1 to 50 GHz) is calibrated in a Coaxial Sensor calibration system. As shown in the figure, especially when more than

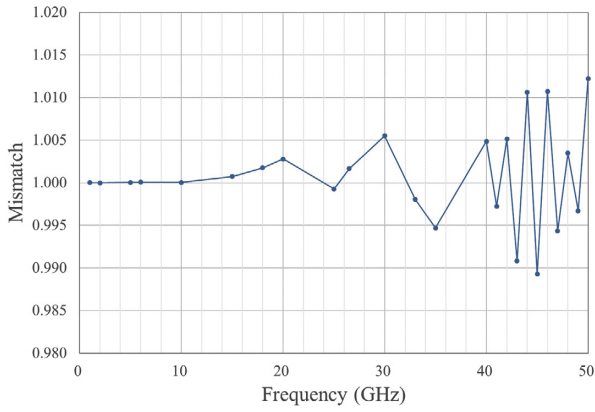


Fig. 7 M graph

Table 1 JCSS certification range

Frequency (GHz)	Calibration and measurement capability (%) (Level of confidence approximately 95%)
0.01	0.60
0.015	0.50
0.02	0.47
0.025, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2	0.46
1.4, 1.5, 1.6, 1.8	0.45
2.0	0.46
2.5, 3.0, 4.0	0.45
5.0	0.47
6.0	0.46
7.0, 8.0	0.65
9.0	0.64
10	0.82
11	0.78
12	0.86
13	1.13
14	1.35
15	1.08
16	1.24
17	1.26
18	1.50

30 GHz, M is far different from 1, and we cannot ignore the effect of M on calibration value (in Fig. 7, at 50 GHz, it has about 1% effect on calibration factor).

4 Traceability

All Power Meter calibrations are traceable at the National Metrology Institute of Japan (NMIJ), which sets Japan’s national standards (secondary standard device). Among those, Power Meters (1 mW, Type-N 50 Ω sensor)

with frequencies from 10 MHz to 18 GHz can be calibrated by JCSS (international MRA compatible) based on ISO/IEC17025[3]–[5]. JCSS is the Measurement Act traceability system based on Japan’s Measurement Act, with the National Institute of Technology and Evaluation managing the registration system for calibration laboratories. These registration criteria are whether there is compliance with the Measurement Act related regulations and items required by ISO/IEC17025.

Table 1 shows the frequencies and calibration and measurement capabilities of JCSS certification. For calibration and measurement capability, it shows the smallest uncertainty value when DUT is calibrated, for uncertainty when reflection Γ_e of DUT sensors is 0; this is written in the registration certificate during JCSS registration.

5 Uncertainty

5.1 Coaxial sensor

Equation (7) shows the propagation equation for uncertainty of simultaneous comparison and substitution method[6]. $u(K_s)$ is the uncertainty of calibration used by an upper-level calibration organization, and $u(x)$ shows the standard uncertainty for x .

$$u_c^2(K_D) = \left(\frac{R_D}{R_S} M\right)^2 u^2(K_S) + \left(K_S \frac{M}{R_S}\right)^2 u^2(R_D) + \left(-K_S \frac{R_D}{R_S^2} M\right)^2 u^2(R_S) + \left(K_S \frac{R_D}{R_S}\right)^2 u^2(M) + s^2(K_D) \tag{7}$$

where, the STD calibration factor $K_S \approx 1$, and due to the power splitter,

$$R_S = P_{MS} / P_{RS} \approx 1, R_D = P_{MD} / P_{RD} \approx 1$$

Moreover, if the calibration factor is calculated with M equal to 1 (corrections are not made using M), then the estimates of K_S , R_D , R_S and M in Equation (7) are all 1, so the sensitivity coefficient for each factor of uncertainty is 1 or -1, which are 1 when squared, so the relative standard uncertainty is obtained using the following equation.

$$\frac{u(K_D)}{K_D} = \sqrt{\left(\frac{u(K_S)}{K_S}\right)^2 + \left(\frac{u(R_D)}{R_D}\right)^2 + \left(\frac{u(R_S)}{R_S}\right)^2 + \left(\frac{u(M)}{M}\right)^2 + \left(\frac{s(K_D)}{K_D}\right)^2} \tag{8}$$

In the case where corrections are not made using M , the factors of uncertainty are (1) Uncertainty of STD, (2) Difference of ambient temperature during DUT calibration vs. during STD calibration conducted by an upper-level organization, (3) Change over time of STD, (4) Number of digits displayed of DUT, (5) Number of digits displayed of STD, (6) Misalignment between DUT and power splitter and between STD and power splitter, (7) Measurement

Table 2 Uncertainty budget example (100 MHz frequency) (No correction)

Factor of uncertainty		Uncertainty	Distribution	Divisor	Standard uncertainty $u(x)$	Sensitivity coefficient $c(x)$	Contribution to uncertainty $ c(x) u(x)$	
K_S	Upper-level calibration	0.9962	0.40%	Normal	2	0.0020	1	0.00199
	Temperature change	0.24%	0.24%	Normal	1	0.0024	1	0.00237
	Change over time	0	0	Normal	1	0	1	0.00000
R_D	DUT resolution	1.0072	0.001	Uniform	$\sqrt{3}$	0.0006	1	0.00058
R_S	STD resolution	1.0189	0.001	Uniform	$\sqrt{3}$	0.0006	-1	0.00058
M	Mismatch	1	0.0002	U	$\sqrt{2}$	0.0001	1	0.00011
$s(K_D)$	Reproducibility	Measure 5 times	0.0010	Normal	$\sqrt{5}$	0.0004	1	0.00045
Combined Standard Uncertainty							0.00324	
Expanded relative uncertainty($k=2$)							0.00647	
Frequency 0.1 GHz, $K_D=0.9848$							0.66 %	

variability.

(1) uses the value (differs by frequency) shown in the calibration certification of STD (normal distribution). (2) is obtained by actual measurements of the effect on calibration value due to the difference between the temperature shown in the calibration certification of STD ($23 \pm 1^\circ\text{C}$), and the ambient temperature during calibration ($23 \pm 2^\circ\text{C}$) (normal distribution). (3) is obtained from fluctuations in calibration value measured by an upper-level calibration organization over a one-year period (normal distribution). (4) and (5) are determined from the digits read of DUT and STD during calibration (round to the nearest 4th decimal) (uniform distribution). (6) is calculated from actual measurements of the reflection coefficients of STD and DUT and the S parameter of Test Port #2 (U distribution). In (7), calibration is repeated n times, and the variability is calculated. However, the surface joining STD and Test Port #2 is taken as a fixed size, while the position of circumference direction of the surface joining DUT and Test Port #2 divided into n pieces (Type-N 50 Ω connector's female core conductor is divided an even number (4 or 6) of times[7], so it is desirable that n is 3 or a higher odd number) is measured repeatedly over the entire circumference ($360/n$ degrees \times n times), and the variability is measured (normal distribution).

Generally, sensors are designed so the reflection coefficients Γ_D and Γ_S are sufficiently small. And from Equation (6), we know that Γ_{g2} is also sufficiently small, so

$$|\Gamma_{g2}\Gamma_D| \ll 1 \quad \text{and} \quad |\Gamma_{g2}\Gamma_S| \ll 1 \quad (9)$$

are true. Therefore, Equation (5) becomes

$$M = \left| \frac{1 - \Gamma_{g2}\Gamma_D}{1 - \Gamma_{g2}\Gamma_S} \right|^2 \approx \left\{ 1 \pm 2 \left(|\Gamma_{g2}\Gamma_D| + |\Gamma_{g2}\Gamma_S| \right) \right\} \quad (10)$$

Therefore, uncertainty generated when M is approximately 1 is expressed by the following equation.

$$u(M) = \frac{2}{\sqrt{2}} \sqrt{|\Gamma_{g2}\Gamma_D|^2 + |\Gamma_{g2}\Gamma_S|^2} \quad (11)$$

Table 2 shows an example (100 MHz frequency) of an uncertainty budget of calibration, from the factors listed above, in the case where correction due to M is not done

Also, Equation (7) is used to obtain the uncertainty in the case where correction due to M is done[8]. In this case, uncertainty of M differs from Equation (11); it adds uncertainty of the measurements Γ_{g2} , Γ_S , and Γ_D :

$$u^2(M) = 4[\Gamma_S - \Gamma_D]^2 u^2(\Gamma_{g2})^2 + 4|\Gamma_{g2}|^2 \left\{ u^2(\Gamma_S) - u(\Gamma_D) \right\} \quad (12)$$

Also, from Equation (6), we get:

$$u^2(\Gamma_{g2}) = u^2(S_{22}) + \left| \frac{S_{21}}{S_{31}} \right|^2 u^2(S_{32}) + \left| \frac{S_{32}}{S_{31}} \right|^2 u^2(S_{21}) + \left| \frac{S_{32}S_{21}}{S_{31}^2} \right|^2 u^2(S_{31}) \quad (13)$$

Here, $u(S_{xx})$ is the standard uncertainty when S_{xx} is measured. The value of uncertainty is calculated using measurements of S parameters, by a calculation formula of the company that manufactures the Vector Network Analyzer (VNA).

As shown in Fig. 7, especially when exceeding 30 GHz, M is far different from 1, so in the case of Power Meters using Coaxial Sensors, NICT currently does correction using M at 30 GHz or greater. Table 3 and 4 show examples

Table 3 Uncertainty budget example (50 GHz frequency) (With correction)

Factor of uncertainty		Uncertainty	Distribution	Divisor	Standard uncertainty u(x)	Sensitivity coefficient c(x)	Contribution to uncertainty c(x) u(x)	
K_S	Upper-level calibration	0.868	3.1%	Normal	2	0.0135	1.01	0.0136
	Temperature change	0.24%	0.24%	Normal	1	0.0021	1.01	0.0021
	Change over time	0	0	Normal	1	0	1.01	0.0000
R_D	DUT resolution	1.0580	0.001	Uniform	$\sqrt{3}$	0.0006	0.83	0.0005
R_S	STD resolution	1.0631	0.001	Uniform	$\sqrt{3}$	0.0006	-0.81	0.0005
M	Mismatch	1.0122	0.0037	U	$\sqrt{2}$	0.0026	0.86	0.0022
$s(K_D)$	Reproducibility	Measure 5 times	0.0010	Normal	$\sqrt{5}$	0.0004	1.00	0.0004
Combined Standard Uncertainty							0.0139	
Expanded relative uncertainty(k= 2)							0.0278	
Frequency 50 GHz, $K_D=0.874$							3.2 %	

Table 4 Uncertainty budget example (100 GHz frequency)

Factor of uncertainty		Uncertainty	Distribution	Divisor	Standard uncertainty u(x)	Sensitivity coefficient c(x)	Contribution to uncertainty c(x) u(x)	
K_S	Upper-level calibration	0.918	3.6%	Normal	2	0.0165	0.94	0.0155
	Temperature change	0.24%	0.24%	Normal	1	0.0022	0.94	0.0020
	Change over time	0	0	Normal	1	0	0.94	0.0000
R_D	DUT resolution	0.3493	0.001	Uniform	$\sqrt{3}$	0.0006	2.46	0.0014
R_S	STD resolution	0.3736	0.001	Uniform	$\sqrt{3}$	0.0006	-2.30	0.0013
M	Mismatch	1	0.0004	U	$\sqrt{2}$	0.0003	0.86	0.0003
$s(K_D)$	Reproducibility	Measure 4 times	0.0010	Normal	2	0.0005	1.00	0.0005
Combined Standard Uncertainty							0.0157	
Expanded relative uncertainty(k= 2)							0.0314	
Frequency 100 GHz, $K_D=0.858$							3.7 %	

of the uncertainty budget of calibration when M is corrected.

5.2 Waveguide sensor

Also in the case of a Waveguide Sensor, like in the case of Coaxial Sensors, Equation (7) is used to obtain uncertainty.

Also, for Equation (6), in cases where

$$|S_{22}| \gg \left| S_{32} \frac{S_{21}}{S_{31}} \right| \tag{14}$$

We focus on ability to approximate

$$\Gamma_{g2} \approx S_{22} \tag{15}$$

As shown in Fig. 6, in a calibration system for Waveguide Sensors, signals output from the multiplier are input into the directional coupler. The directional coupler’s degree of coupling is 6 dB (uniform value), so the S parameter (S_{31}) between Port #1 and Reference Port #3 becomes approximately -6 dB. On the other hand, two isolators are inserted between Port #1 and Test Port #2, and due to the loss when passing through these, and the loss when passing through the directional coupler, the S parameter (S_{21}) between Port #1 and Test Port #2 becomes approximately -6 dB

Also, considering the S parameter between Test Port #2 and Reference Port #3, there is 40 dB isolation due to two isolators, and the directionality of the directional coupler used ($-20\log|S_{31}/S_{21}|$) is 20 dB or greater. Therefore, S_{32} becomes approximately 60 dB smaller than the reflection of Test Port #2 (S_{22}).

Therefore, in the calibration system shown in Fig. 6, the relationship of Equation (14) holds true, so we can use the Equation (15) approximation formula. Thus it is not necessary to use VNA to measure all S parameters of the calibration system. So we can obtain Γ_{g2} by only measuring (possible by 2-Port VNA) the reflection coefficient of Test Port #2 (S_{22}). The factors of uncertainty are similar to those for calibration of Coaxial Sensors, but variability of measurements are evaluated by measuring n times (n is an even number) with the direction faced (waveguide) changed 180 degrees for Test Port #2 and DUT. Table 4 shows an example of an uncertainty budget of calibration (100 GHz frequency).

6 Changes over years

In order to check the calibration system and validity of calibration results, the same calibration of DUT is done each year, and changes in calibration results over the years are evaluated.

Figure 8 shows Power Meter calibration results from 2011 to 2014 (Type-N 50 Ω sensors, 10 MHz to 18 GHz frequency). Figure 9 shows their calibration results (W-Band) from 2012 to 2016. However, frequencies that were not calibrated are not marked.

It was decided to evaluate changes over the years by the number of E_n [9]. The number of E_n is used in evaluations such as round robin tests. E_n is expressed by the following equation.

$$E_n = \frac{LAB - REF}{\sqrt{U_{LAB}^2 + U_{REF}^2}} \quad (16)$$

where,

LAB: Measurement value from a participating calibration organization

REF: Measurement value from a reference calibration organization

U_{LAB} : Expanded uncertainty from a participating calibration organization ($k = 2$)

U_{REF} : Expanded uncertainty from a reference calibration organization ($k = 2$)

and the evaluation is satisfactory if $|E_n| \leq 1$, not satisfac-

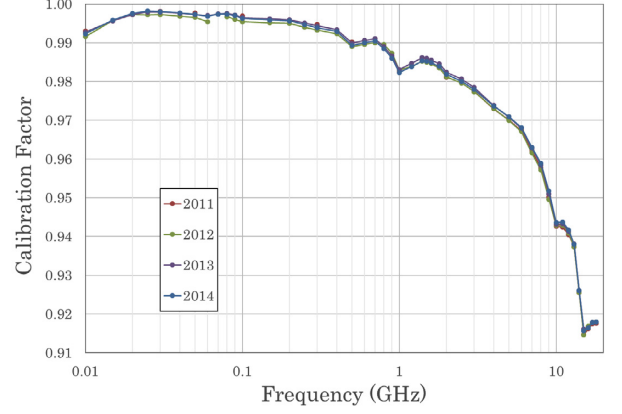


Fig. 8 Same DUT calibration results (10 MHz to 18 GHz)

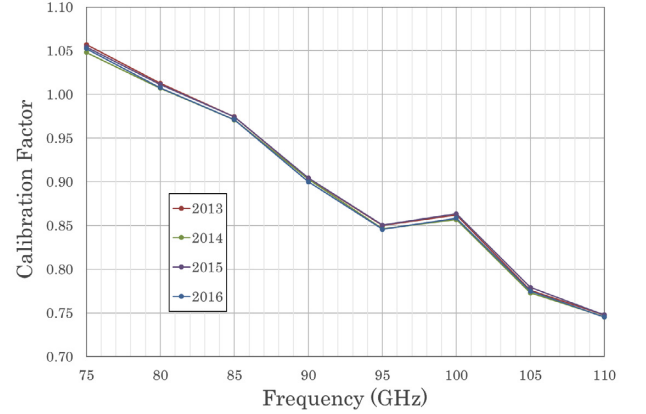


Fig. 9 Same DUT calibration results (75 to 110 GHz)

tory if $|E_n| > 1$.

Evaluations were performed using Equation (16) with the calibration value of the reference year (final year) as REF (uncertainty U_{REF}), and each year's calibration value as LAB (uncertainty U_{LAB}). The number of E_n is obtained for each frequency, and the highest value over the past four years is obtained. However, the calibration value of STD by an upper-level calibration organization differs slightly each year, but only the calibration results are evaluated, so the ratio vs. the reference year (final year's STD calibration value/each year's STD calibration value) is multiplied by each calibration value, and the STD change portion is removed.

The evaluation results are that the maximum number of E_n is 0.24 for Power Meters using Type-N 50 Ω sensors (10 MHz to 18 GHz frequency), 0.15 E_n for 2.4 mm sensors (1 to 50 GHz frequency), 0.31 E_n for V-Band sensors, and 0.11 E_n for W-Band sensors. All are evaluated as satisfactory.

As described above, calibration results over the past four years by this system using the simultaneous comparison and substitution method show that the maximum number of E_n within the evaluation range is less than 1, and stable calibration results are obtained.

7 Conclusion

This described a method for calibration of Power Meters until 110 GHz frequency, a calibration system, and a method for calculating uncertainty, that enable very precise calibration using the simultaneous comparison and substitution method. This also showed that stable results were obtained for calibration of Power Meters by this calibration system. Representative expanded uncertainties (coverage factor $k = 2$) were 0.66% for 100 MHz, 3.2% for 50 GHz, and 3.7% for 100 GHz.

The method of Power Meter calibration using the simultaneous comparison and substitution method can also be applied to absolute values of calibration of spectrum analyzers, and is actually being used in calibrations.

Appendix. Derivation of Equation (2)

In the calibration system with simultaneous comparison and substitution method shown in Fig. 2, when a standard device is connected to Test Port #2, and it is expressed using S parameters, the following equation is obtained.

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{A.1}$$

$$a_1 = a_G + \Gamma_G b_1 \tag{A.2}$$

$$a_2 = \Gamma_S b_2 \tag{A.3}$$

$$a_3 = \Gamma_R b_3 \tag{A.4}$$

Here, the S matrix expresses characteristics of a three port circuit shown by the dashed lines containing the power splitter. a_G is the source power of signal source, Γ_G is the reflection coefficient of signal source, Γ_R is the reflection coefficient of reference device, and Γ_S is the reflection coefficient of the standard device. From these equations, the power P_{in}^{STD} input into the standard device, and the power P_{in}^{RS} input into the reference device, are each obtained by

$$P_{in}^{STD} = |b_2|^2 = \left| \frac{D_{(2S1)(11)}}{D} \right|^2 |a_G|^2 \tag{A.5}$$

$$P_{in}^{RS} = |b_3|^2 = \left| \frac{D_{(3S1)(11)}}{D} \right|^2 |a_G|^2 \tag{A.6}$$

where:

$$D = \det \begin{bmatrix} 1 - S_{11}\Gamma_G & -S_{12}\Gamma_S & -S_{13}\Gamma_R \\ -S_{21}\Gamma_G & 1 - S_{22}\Gamma_S & -S_{23}\Gamma_R \\ -S_{31}\Gamma_G & -S_{32}\Gamma_S & 1 - S_{33}\Gamma_R \end{bmatrix} \tag{A.7}$$

$$D_{(2S1)(11)} = \det \begin{bmatrix} S_{21} & -S_{23}\Gamma_R \\ S_{31} & 1 - S_{33}\Gamma_R \end{bmatrix} = S_{21} \left\{ 1 - \left(S_{33} - S_{23} \frac{S_{31}}{S_{21}} \right) \Gamma_R \right\} \tag{A.8}$$

$$D_{(3S1)(11)} = \det \begin{bmatrix} 1 - S_{22}\Gamma_S & S_{21} \\ -S_{32}\Gamma_S & S_{31} \end{bmatrix} = S_{31} \left\{ 1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_S \right\} \tag{A.9}$$

Here, $\det[A]$ expresses the matrix formula of matrix A. Now, if we simultaneously measure two incident powers and obtain their ratio, then by (A.5) and (A.6), we obtain:

$$R_S = \frac{P_{MS}}{P_{RS}} = \frac{K_S}{K_{RS}} \frac{P_{in}^{STD}}{P_{in}^{RS}} = \frac{K_S}{K_{RS}} \left| \frac{D_{(2S1)(11)}}{D_{(3S1)(11)}} \right|^2 = \frac{K_S}{K_{RS}} \left| \frac{S_{21}}{S_{31}} \right|^2 \frac{\left| 1 - \left(S_{33} - S_{23} \frac{S_{31}}{S_{21}} \right) \Gamma_R \right|^2}{\left| 1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_S \right|^2} \tag{A.10}$$

Here, the relationship between $P_{in}^{STD} = P_{MS}/K_S$ and $P_{in}^{RS} = P_{RS}/K_R$ was used. K_S and K_R are each the calibration factors of the standard device and reference device.

Next, if instead of the standard device, a device under test (DUT) is connected to Test Port #2, then measuring the input power obtains:

$$R_D = \frac{P_{MD}}{P_{RD}} = \frac{K_D}{K_R} \frac{P_{in}^{DUT}}{P_{in}^{RD}} = \frac{K_D}{K_R} \left| \frac{S_{21}}{S_{31}} \right|^2 \frac{\left| 1 - \left(S_{33} - S_{23} \frac{S_{31}}{S_{21}} \right) \Gamma_R \right|^2}{\left| 1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_D \right|^2} \tag{A.11}$$

Here, Γ_D is the reflection coefficient of the device under test (DUT), where the relationships of $P_D = P_{MD}/K_D$ and $P_{in}^{RD} = P_{RD}/K_R$ were used. K_D is the calibration factor of the device under test.

Now, if we calculate the ratio of Equation (A.10) and Equation (A.11), we get

$$\frac{R_S}{R_D} = \frac{K_S}{K_D} \left| \frac{1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_D}{1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_S} \right|^2 \tag{A.12}$$

If we transform the equation, we obtain the following Equation (2).

$$K_D = K_S \frac{R_D}{R_S} \left| \frac{1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_D}{1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_S} \right|^2 \quad (\text{A.13})$$

In the equation's derivation process, the power splitter's S_{21} and S_{31} , the calibration factor K_R and reflection coefficient Γ_R of the reference device, and matrix formula D shown in Equation (A.7), are all deleted, so there is no need to actually obtain them.

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Kouichi SEBATA

Senior Researcher, Electromagnetic Compatibility Laboratory, Applied Electromagnetic Research Institute Calibration of Measuring Instruments and Antennas for Radio Equipment, geodesy



Iwao NISHIYAMA

Electromagnetic Compatibility Laboratory, Applied Electromagnetic Research Institute Calibration of Measuring Instruments and Antennas for Radio Equipment



Katsumi FUJII, Dr. Eng.

Research Manager, Electromagnetic Compatibility Laboratory, Applied Electromagnetic Research Institute Calibration of Measuring Instruments and Antennas for Radio Equipment, Electromagnetic Compatibility



Tsutomu SUGIYAMA

Senior Researcher, Electromagnetic Compatibility Laboratory, Applied Electromagnetic Research Institute Calibration of Measuring Instruments and Antennas for Radio Equipment



Kojiro SAKAI

Technical Expert, Electromagnetic Compatibility Laboratory, Applied Electromagnetic Research Institute Calibration of Measuring Instruments and Antennas for Radio Equipment