

2-2-3 Power Meter Calibration for 10 W (10 W, 50 Ω /75 Ω)

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NICT performs calibration services for power meters at 10 W in the frequency range from 100 kHz to 9 GHz for sensors with a Type-N 50 ohm connector and from 1 MHz to 1 GHz for sensors with a Type-N 75 ohm connector. The calibration of 10 W sensors uses dedicated directional couplers in order to achieve high-accuracy of the calibration and to keep the traceability to the national standards that provide calibration services only for 1mW-sensors.

1 Introduction

NICT has been performing calibration of high frequency power meters capable of measuring electric power of 1 mW. In addition, we also calibrate power meters capable of measuring 10 W in order to deal with aerial power of several watts to several tens of kilowatts, which is used by radio stations in real life.

Conventionally, calibration of high frequency power meters at 10 W had been performed in the following manner. Regarding power meters equipped with a 50 Ω input impedance sensor that has a type-N connector input terminal with 50 Ω characteristic impedance (hereinafter referred to as 50 Ω power meters), their calibration had been performed in the frequency range of 1 MHz to 2 GHz (10 MHz to 2 GHz for As a JCSS accredited calibration laboratory). On the other hand, regarding power meters equipped with a 75 Ω input impedance sensor that has a type-N connector input terminal with 75 Ω characteristic impedance (hereinafter referred to as 75 Ω power meters), their calibration had been conducted in the frequency range of 1 to 500 MHz [1].

However, radio waves in a higher frequency range have been used in recent years to meet the demand for increased volume and speed of communication, as the popularity of mobile terminals, represented by smartphones and wireless LAN, increased rapidly. At present, frequencies as high as 5.7 GHz are used [2][3]. Because SAR measurements are required to obtain a license for mobile terminals [4][5], and the use of even higher frequency ranges is under consideration, it has become necessary to increase calibration frequencies for 50 Ω power meters.

Moreover, in relation to cables and connectors with 75 Ω characteristic impedance, which have been used for

TV broadcasting, the broadcast format shifted from analog to digital terrestrial television broadcasting. Consequently, the use of VHF frequencies (90–222 MHz) was terminated and UHF frequencies (470–710 MHz) replaced them, and therefore, the demand for calibration of 75 Ω power meters at higher frequencies has increased.

In consideration of such circumstances, and to meet the demand for calibrating power meters capable of measuring 10 W, NICT developed a new calibration system in order to expand the range of calibration frequencies, increase calibration accuracy, and reduce the magnitude of calibration uncertainty. Then we performed calibration of 50 Ω power meters at 100 kHz to 9 GHz (As a JCSS accredited calibration laboratory, NICT performs this at 10 MHz to 9 GHz) and calibration of 75 Ω power meters at 1 MHz to 1 GHz.

In this paper, we present the calibration method and system we developed and explain methods to evaluate uncertainty associated with calibration results.

2 Power meter used to measure 10 W

As shown in Fig. 1, a terminal type high frequency power meter capable of measuring electric power of 10 W consists of an indicator part, a sensor, and a detachable 30 dB fixed attenuator (ATT). The indicator part and the sensor are connected using a special cable.

The indicator is equipped with a reference signal source (type-N 50 Ω (f) connector), which outputs 50 MHz, 1 mW signals. When using the power meter, a calibration practitioner should detach the ATT beforehand, attach the sensor to the reference signal source, and perform calibration at 1 mW (CAL). Because the accuracy of 50 MHz, 1 mW output signals from the reference signal source influences

calibration results, NICT calibrates a complete set of the indicator part, sensor, and ATT. When the 75 Ω power meter is used, the type-N 50 Ω (f) connector, which is attached to the reference signal source, cannot be connected to the sensor directly, as it is attached to an incom-

patible type-N 75 Ω (m) connector. Accordingly, it is necessary to use a 75 Ω/50 Ω conversion adapter to connect the two ends before performing CAL. When one purchases the sensor, this adapter usually comes with the sensor as an accessory item. Conversion between 50 and 75 Ω causes a passing loss of 0.1 to 0.2 dB. However, due to differences in the conversion losses of adapters, NICT performs calibration on the complete set described above.

3 Calibration factor

Characteristics of power meters used to measure 10 W can be expressed in terms of calibration factor K . As shown in Fig. 1, when incident power P_{in} enters the power meter, an indicator value P_M is displayed on the indicator part. The following relationship exists between the two variables.

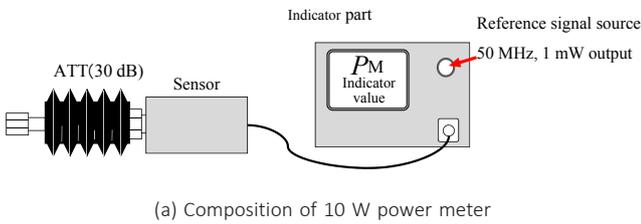
$$K = \frac{P_M}{P_{in}} \tag{1}$$

Note that incident power P_{in} is different from input power (which is the quantity obtained by subtracting reflected power from incident power, that is, power consumed by the power meter).

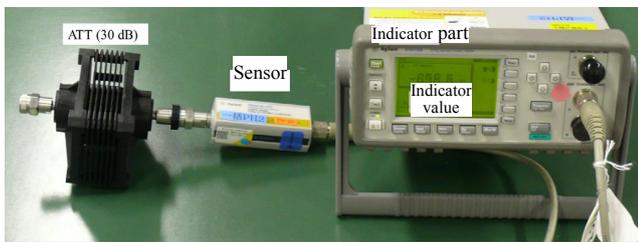
4 Calibration methods

4.1 Simultaneous comparison method

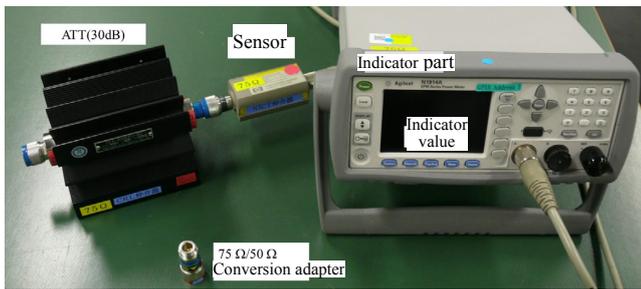
Methods to calibrate power meters include the comparison method, simultaneous comparison method, and simultaneous comparison and substitution method [6]. NICT has adopted the simultaneous comparison method,



(a) Composition of 10 W power meter



(b) 50 Ω power meter



(c) 75 Ω power meter

Fig. 1 Power meter used to measure 10 W

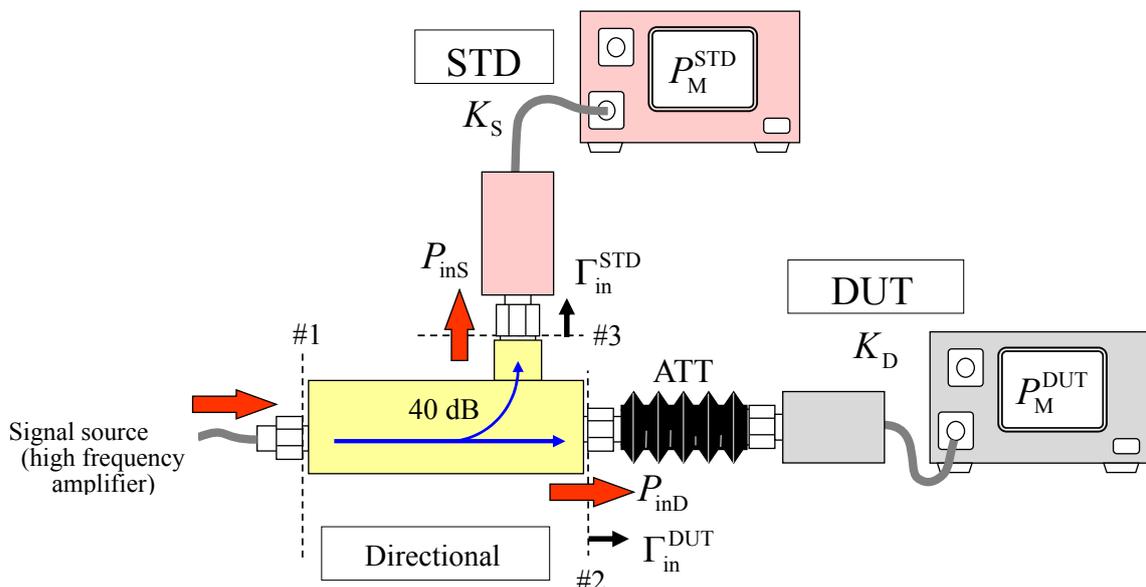


Fig. 2 Simultaneous comparison method using a directional coupler

which involves the use of a directional coupler for the purpose of calibrating power meters that are used to measure 10 W [7].

The concept of the method is illustrated in Fig. 2. The power output from the signal source (high frequency amplifier) enters the directional coupler from port #1 and then is distributed to port #2 (P_{inS}), which is connected to a device under test (DUT), and port #3 (P_{inD}), which is connected to a standard device (STD), serving as a calibration reference. Given that the coupling factor of the directional coupler is about 40 dB, P_{inS} becomes about 1 mW (0 dBm) when P_{inD} is 10 W (+40 dBm). One can determine the calibration factor of DUT by correcting the coupling factor and an insertion loss associated with the directional coupler using the ratio of power values displayed on the indicator part.

The coupling factor and a insertion loss associated with the directional coupler should be accurately measured beforehand to obtain S parameters, using a vector network analyzer (VNA). The relationships between the coupling factor and S parameters, and between a insertion loss and S parameters can be expressed as $-20\log_{10}|S_{31}|[\text{dB}]$ and $-20\log_{10}|S_{21}|[\text{dB}]$, respectively.

The simultaneous comparison method uses a ratio of two values measured simultaneously using a DUT and STD. As such, change in power output from a signal source (high frequency amplifier) does not affect calibration result. Also, as described below, equations used for calibration, including the uncertainty due to mismatch, do not include a reflection coefficient of a signal source (high frequency amplifier). It is difficult to measure a reflection coefficient when a signal source, particularly a high frequency amplifier, is in action. Thus, it is very advantageous that the calibration of a 10 W power meter does not require the coefficient.

4.2 Calibration procedure

Calibration is conducted in the following steps.

(1) Measurement of reflection coefficients

Using a VNA, measure a reflection coefficient for the STD sensor ($\Gamma_{\text{in}}^{\text{STD}}$). Also, measure a reflection coefficient for the input terminal of the ATT ($\Gamma_{\text{in}}^{\text{DUT}}$) when the ATT is connected to the DUT sensor (see Fig. 2). These measurement results are not needed to determine a calibration factor, but they are needed to estimate calibration uncertainty. Also, measurement of reflection coefficients before performing calibration allows a calibration practitioner to detect any malfunctions or defects in the system in advance.

(2) Measuring characteristics of a calibration system

Using a VNA, measure the S parameters of the directional coupler, and calculate the equivalent signal source reflection coefficients (Γ_{g2} , Γ_{g3}). The two coefficients can be determined using the S parameters of the directional coupler measured and the following equations.

$$\Gamma_{g2} = S_{22} - S_{32} \frac{S_{21}}{S_{31}} \quad (2)$$

$$\Gamma_{g3} = S_{33} - S_{23} \frac{S_{31}}{S_{21}} \quad (3)$$

Again, although these values are not necessary to determine a calibration factor, in addition to the reflection coefficients measured in step (1), they are needed to estimate uncertainty due to mismatch.

(3) Calibration of DUT

Attach the DUT and STD to ports #2 and #3, respectively, of the directional coupler. Adjust power output from the signal source so that incident power (P_{inD}) to the DUT matches the predetermined level (10 W) while taking account of insertion loss and the coupling factor (approx. 40 dB) of the directional coupler. When the predetermined power level is achieved, measure the indicator values of the DUT P_M^{DUT} and STD P_M^{STD} at the same time.

Let 100 repetitions of measurements comprise a set. Measurements on a DUT can be made using either a 50 or 75 Ω power meter. When a 50 Ω power meter is used, take a set of DUT measurements while rotating the DUT sensor 72 degrees at a time. Thus, a total of five sets of measurements are taken per DUT at the 0, 72, 144, 216, and 288 degree rotational positions. When a 75 Ω power meter is used, take a set of DUT measurements while rotating the DUT sensor 90 degrees at a time. Thus, a total of four sets of measurements are taken per DUT at the 0, 90, 180, and 270 degree rotational positions. Using the measurement data, calculate the ratio of P_M^{DUT} to P_M^{STD} and then the average of $(P_M^{\text{DUT}}/P_M^{\text{STD}})$.

(4) Determining two calibration factors

Determine the calibration factor of the DUT, K_D , by plugging the average of $(P_M^{\text{DUT}}/P_M^{\text{STD}})$ into Equation (4), where K_S denotes the calibration factor of the STD.

$$K_D = K_S \left| \frac{S_{31}}{S_{21}} \right|^2 \left(\frac{P_M^{\text{DUT}}}{P_M^{\text{STD}}} \right) M \quad (4)$$

Then, determine M parameters, which represent mismatch occurring between the directional coupler and the STD/

DUT using Equation (5).

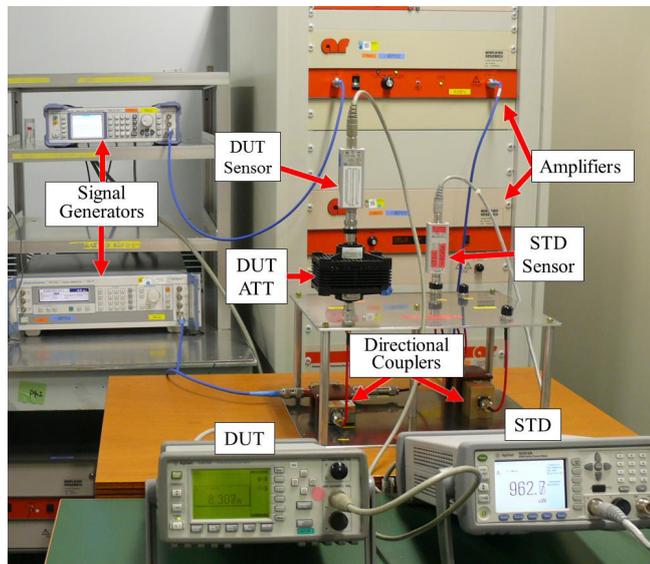
$$M = \left| \frac{1 - \Gamma_{g2} \Gamma_{in}^{DUT}}{1 - \Gamma_{g3} \Gamma_{in}^{STD}} \right|^2 \quad (5)$$

When no mismatch occurs, M takes the value of 1 and does not influence calibration results.

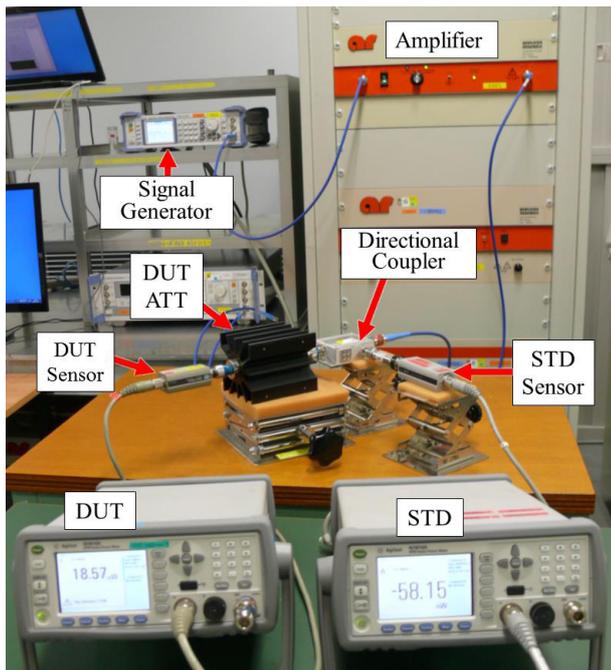
4.3 Calibration system

4.3.1 Calibration system for a 50 Ω power meter

Figure 3 (a) shows a system used to calibrate a 50 Ω power meter. This system—equipped with two signal



(a) Calibration system for 50 Ω power meter



(b) Calibration system for 75 Ω power meter

Fig. 3 Calibration system for power meter measuring 10 W

generators, three high frequency amplifiers, and two directional couplers—enables calibration of a 50 Ω power meter measuring 10 W in the frequency range between 100 kHz and 9 GHz. The system uses the simultaneous comparison method explained in 4.1 above. Specific devices used in the system are listed in Table 1 (a).

In this calibration system, when incident power from port #2 to the DUT is 10 W, incident power from port #3 to the STD can be set around 1 mW by adjusting the coupling factor of the directional coupler to about 40 dB. This feature allows the use of a power meter, which had been calibrated at 1 mW, as an STD. Moreover, in the system, connectors to ports #2 and #3 are facing upward. This arrangement allows uniform contact between port and connector surfaces, and easy measurements while rotating the DUT sensor 72 degrees at a time. When we attach connectors to the ports and tighten, we always apply the same torque to the connectors using a torque wrench, so we can achieve a good connection every time. The calibration procedure using this system is the same as the steps explained in 4.2.

4.3.2 Calibration system for a 75 Ω power meter

Figure 3 (b) shows a system used to calibrate a 75 Ω power meter. This system— equipped with one signal generator, high frequency amplifier, and directional coupler—enables the calibration of a 75 Ω power meter measuring 10 W in the frequency range between 1 MHz and 1 GHz. The system uses the simultaneous comparison method explained in 4.1 above. Specific devices used in the system are listed in Table 1 (b). This system is applicable in a narrower frequency range compared to the calibration system for 50 Ω power meters, because the directional coupler compatible with the former system works in a limited frequency range. It is possible to expand the frequency range of the system if the system can integrate a directional coupler, which works in a frequency range lower than 1 MHz or higher than 1 GHz.

In this calibration system, port #1 of the directional coupler, port #2 to which a DUT is connected, and port #3 to which the STD is connected have characteristic impedances of 75 Ω, 75 Ω, and 50 Ω, respectively. Another characteristic of the system is that it employs a directional coupler whose coupling factor is set at about 40 dB. However, the directional coupler we used had a coupling factor of about 30 dB ($|S_{31}| \approx 30$ dB) and port #3 had a large reflection coefficient (S_{33}). We resolved these two issues and obtained acoupling factor of about 40 dB by

Table 1 Devices used in calibration
(a) Calibration system for 50 Ω power meter

Device	Frequency range	Manufacturer	Model
Signal generator	100 kHz ~ 1 GHz	Rohde & Schwartz	SMB100A
	1.2 GHz ~ 9 GHz	Rohde & Schwartz	SMR60
Amplifier	100 kHz ~ 1 GHz	Amplifier Research	50WD1000
	1.2 GHz ~ 3 GHz	Amplifier Research	50S1G4A
	4 GHz ~ 9 GHz	Amplifier Research	20S4G11AG4A
Directional coupler	100 kHz ~ 1 GHz	WERLATONE	C8445-10
	1.2 GHz ~ 9 GHz	Agilent Technologies	773D

(b) Calibration system for 75 Ω power meter

Device	Frequency range	Manufacturer	Model
Signal generator	9 kHz ~ 1.1 GHz	Rohde & Schwartz	SMB100A
Amplifier	DC ~ 1000 MHz	Amplifier Research	50WD1000
Directional coupler	1 MHz ~ 1000MHz	R&K	DC001M102-3040

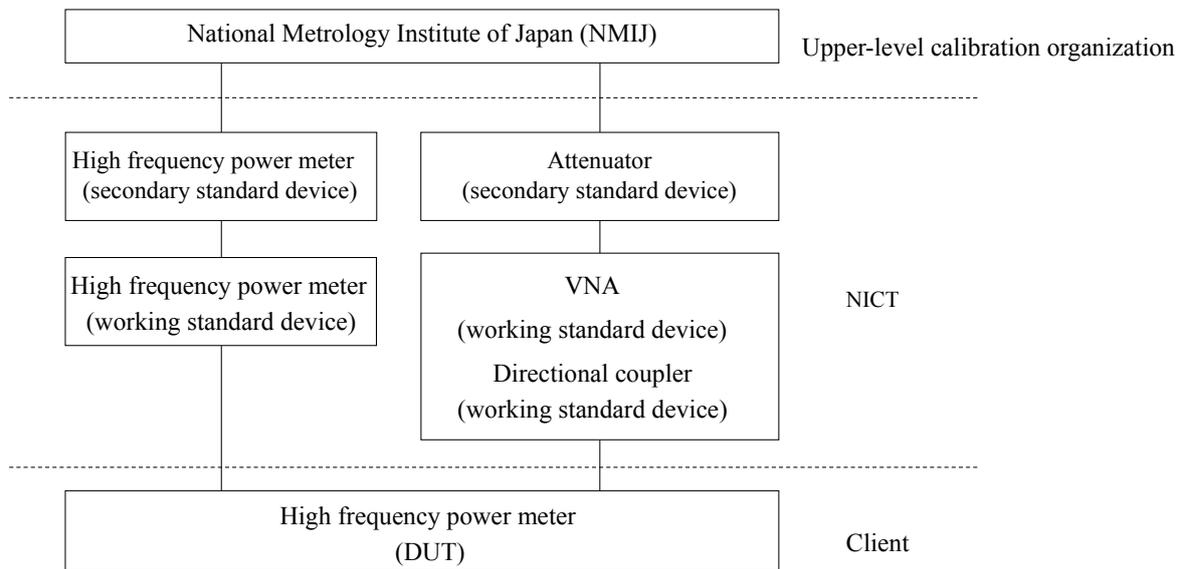


Fig. 4 Traceability chart

connecting a 10 dB ATT with excellent reflection characteristics to port #3. With this adjustment, the system has become capable of calibrating 75 Ω power meters using an STD power meter with a type-N 50 Ω connector, which had been calibrated at 1 mW. Calibration procedure using this system is same as the steps explained in 4.2.

Another issue with this system was that input port #1 of the directional coupler had characteristic impedance of 75 Ω. Consequently, this port could not be connected directly to a signal source (high frequency amplifier) that has an output port with its characteristic impedance being 50 Ω. An option to resolve this issue is to use a 75 Ω/50 Ω conversion adapter. However, we dealt with this issue by attaching an adapter, which can change the diameter of the

internal conductor, to the signal source (high frequency amplifier). When using this adapter, the reflection coefficient, which applies to the direction from port #1 to the signal source, can be calculated using Equation (6).

$$|\Gamma_G| = \left| \frac{50 - 75}{50 + 75} \right| = 0.2 \quad (6)$$

The reflection coefficient can be converted into the voltage standing wave ratio (VSWR) of 1.5. If a 75 Ω power meter is calibrated by attaching it directly to this connector, the calibration will be greatly affected by reflection. However, as described in 4.1, the reflection coefficient of a signal generator (high frequency amplifier) does not influence calibration results when the simultaneous com-

Table 2 Frequency range in which calibration is performed and the corresponding maximum measurement capacity
(a) 50 Ω power meter

Frequency range (MHz)	Maximum measurement capacity (%)	JCSS registration
0.1	2.9	–
0.3	2.8	–
0.5, 1, 2, 3, 5	2.0	–
10, 15, 20, 25, 30, 40, 50, 60, 70, 80, 90, 100, 150, 200, 250, 300, 400, 500, 600, 700, 800, 900	1.9	JCSS
1000	2.0	JCSS
1200, 1400, 1500, 1600, 1800, 2000, 2500, 3000, 4000, 5000, 6000	2.9	JCSS
7000	3.0	JCSS
8000, 9000	3.1	JCSS

(b) 75 Ω power meter

Frequency range (MHz)	Maximum measurement capacity (%)	JCSS registration
1, 2, 3, 5	2.7	–
10, 15, 20, 25, 30, 40, 50, 60, 70, 80, 90, 100, 150, 200, 250, 300, 400, 500, 600, 700, 800, 900, 1000	2.0	–

parison method is used.

4.4 Traceability

As shown in Fig. 4, all power meter calibrations are traceable at the National Metrology Institute of Japan (NMIJ), which sets Japan's national standards.

Table 2 lists frequency ranges in which calibration can be performed and corresponding maximum measurement capacities. A maximum measurement capacity refers to relative expanded uncertainty when the reflection of DUT (Γ_{in}^{DUT}) = 0. In other words, it is the smallest relative expanded uncertainty associated with calibration using the system described in 4.3. Relative expanded uncertainty is estimated at the confidence level of 95% (coverage factor $k = 2$).

Table 2 (a) lists frequency ranges and corresponding maximum measurement capacities associated with the calibration of 50 Ω power meters. In a JCSS-registered frequency band (10 MHz-9 GHz), NICT provides calibration service based on Japan's Measurement Act (JCSS accredited calibration laboratory) [8], and calibration performed in this manner satisfies ISO/IEC17025 [9]. Such calibration also complies with international agreements (mutual recognition arrangements or MRA) and is considered to be a one-stop testing procedure [10]. Maximum measurement capacities range between 1.9 and 3.1 %, and

the percentage increases with increasing frequency. This relationship exists because mismatch between a directional coupler and power meters (DUT and STD) increases with increasing frequency. We will explain about this from the viewpoint of the uncertainty budget in 6.

Table 2 (b) lists frequency ranges and corresponding maximum measurement capacities associated with the calibration of 75 Ω power meters. All power meter calibrations of 75 Ω power meters are also traceable at the NMIJ. Maximum measurement capacities range between 2.0 and 2.7 %, and increase with increasing frequency largely due to the influence of mismatch, as was the case with the calibration of 50 Ω power meters. Again, we will explain this relationship from the viewpoint of the uncertainty budget in 6.

The issue regarding mismatch may be resolved by replacing the directional coupler used in the system so that the values obtained from Equations (2) and (3) become smaller. By examining these values, we found that the reflection of port #2 of the directional coupler (S_{22}) is sufficiently small, and S_{23} and S_{32} are very small due to separation between ports #2 and #3. These results indicate that good directionality of the coupler is necessary. Also, calibration of both 50 Ω and 75 Ω power meters conducted in frequencies between 100 kHz and 5 MHz resulted in an increase of maximum measurement capacities

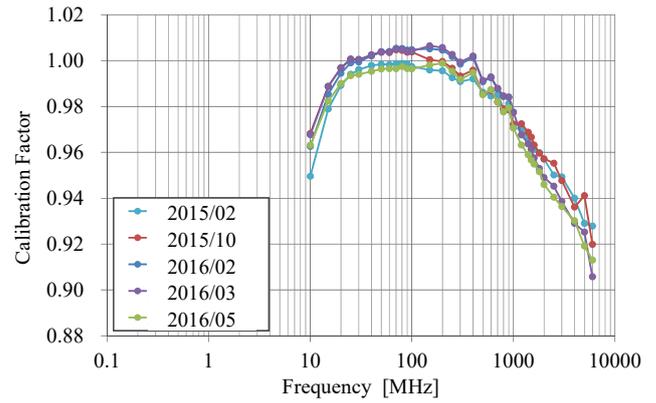
with increasing frequencies. This trend is attributed to a large quantity of uncertainty associated with the calibration factor for STD, which gives a standard value, and was caused largely by uncertainty of standard power meters, which were calibrated through direct comparison with national standards.

5 Examples of calibration results

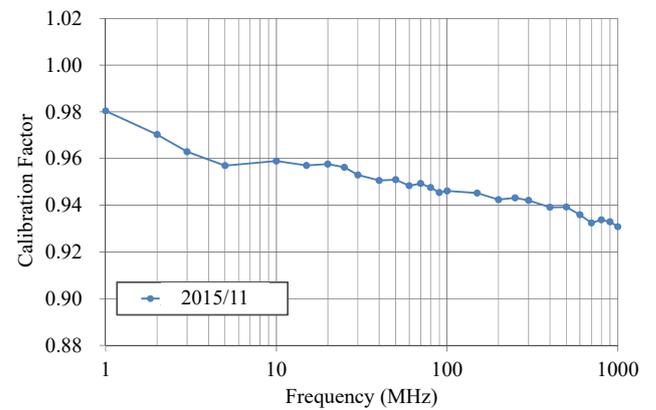
Figure 5 shows results of calibration using the systems described above. Specifically, Fig. 5 (a) shows results (calibration factors) obtained from the calibration of a 50 Ω power meter (used as a DUT), which consisted of components listed in Table 3 (a). The results represent calibrations performed periodically from February 2015 to May 2016. The sensor of the 50 Ω power meter and the 30 dB ATT are sold together as a set. When they were used together and the calibration factor was 1.00, there was a match between values displayed on the indicator part and incident power, which entered the sensor after passing the 30 dB ATT. Variability among calibrations performed five times was within $\pm 1.1\%$, indicating satisfactory repeatability of calibration.

Figure 5 (b) shows results (calibration factors) obtained from the calibration of a 75 Ω power meter, which consisted of the components listed in Table 3 (b). Unlike the 50 Ω power meter, the sensor of the 75 Ω power meter and that of the 30 dB ATT are sold separately. As such, when they were used together, values displayed on the indicator part were about one-thousandth less than incident power that entered the 30 dB ATT due to the attenuation caused by the ATT. To correct this inconsistency, we multiplied the displayed values by 1,000 and used the resulting values as calibration factors. For example, when incident power was 10.00 W and the value displayed on the indicator part was 10.00 mW, we multiplied the displayed value by 1,000 to make the displayed value 10.00 W. Then, we used the calibration factor of 1.000.

The calibration system was developed only recently, so we have performed calibration using the system only once so far (Fig. 5 (b)), and we have not evaluated the repeatability of calibration. We plan to conduct periodic calibrations and determine their variability. We have been calculating calibration factors by plugging 1 into M in Equation (4). The effect of this substitution is interpreted as uncertainty.



(a) 50 Ω power meter



(b) 75 Ω power meter

Fig. 5 Examples of calibration results

Table 3 Manufacturer information of DUT components

(a) 50 Ω power meter

	Manufacturer	Model number
Indicator part	Agilent Technologies	E4419B
Sensor	Agilent Technologies	N8481B
30 dB fixed attenuator		

(b) 75 Ω power meter

	Manufacturer	Model number
Indicator part	Agilent Technologies	E4419B
Sensor	Agilent Technologies	8483A
30 dB fixed attenuator	Tamagawa Electronics	CFA-50NCPJ-30

6 Uncertainty

In calibration results, calibration factors are always associated with uncertainty. The uncertainty represents the collective effects of several uncertain factors in measurements. Uncertainty related to the calibration of power meters is expressed in terms of relative uncertainty. Relative standard uncertainty, resulting from the combined effects

of all uncertainty factors, can be calculated using Equation (7). This equation determines combined relative uncertainties. In the equation, fractions represent different relative standard uncertainty factors, and numbers in brackets before the fractions represent sensitivity coefficients, $c(x_i)$, for different uncertainty factors. A sensitivity coefficient denotes the influence of a given uncertainty factor on calibration results, and can be calculated by partially differentiating Equation (4), which determines a calibration factor [11]. Even if a given uncertainty factor takes a large value, its influence on calibration results is small, if the corresponding sensitivity coefficient takes a small value. Conversely, even if a given uncertainty factor takes a small value, its influence on calibration results is large, as long as the corresponding sensitivity coefficient takes a large value.

$$\frac{u(K_D)}{K_D} = \sqrt{\frac{\begin{aligned} & (+1)^2 \left\{ \frac{u(K_S)}{K_S} \right\}^2 + (+2)^2 \left\{ \frac{u(|S_{31}|)}{|S_{31}|} \right\}^2 + (-2)^2 \left\{ \frac{u(|S_{21}|)}{|S_{21}|} \right\}^2 \\ & + (+1)^2 \left\{ \frac{u(P_M^{DUT})}{P_M^{DUT}} \right\}^2 + (-1)^2 \left\{ \frac{u(P_M^{STD})}{P_M^{STD}} \right\}^2 + (+1)^2 \left\{ \frac{u(M)}{M} \right\}^2 \end{aligned}}{+ (+1)^2 \left\{ \frac{s(K_{DUT})}{K_{DUT}} \right\}^2}} \quad (7)$$

The following eight factors are considered to be the sources of uncertainty when calibrating power meters: (1) uncertainty associated with the calibration factor of STD, (2) uncertainty associated with the measurement of $|S_{31}|$ between ports #1 and #3 of the directional coupler, (3) uncertainty associated with the measurement of $|S_{21}|$ between ports #1 and #2, (4) resolution of STD measurement, (5) resolution of DUT measurement, (6) mismatch between STD/DUT and a directional coupler, (7) variability caused by heated 30 dB ATT of DUT, and (8) measurement variability.

To deal with factor (1), we use calibration uncertainty associated with the power meter of NICT, which is traceable to the national standard (normal distribution). To deal with factors (2) and (3), we use standard uncertainty yielded when $|S_{31}|$ and $|S_{21}|$ of the directional coupler were measured using a VNA (normal distribution). We determine measurement resolutions (4) and (5) based on the number of digits present in numerical values displayed on STD and DUT (uniform distribution). To deal with factor (6), we calculate the S parameters of the directional coupler through actual measurements (U-shaped distribution). To deal with factor (7), we measure change in calibration value due to temperature change when the incident power of 10 W enters the DUT (uniform distribution). And to

deal with factor (8), we take five sets of measurements and calculate the variability of calibration results (normal distribution).

Table 4 shows an example of uncertainty calculation. Calibration factors were determined from the calibration of a 50 Ω power meter at the frequency of 9 GHz (a) and the calibration of a 75 Ω power meter at the frequency of 1 GHz (b). These tables revealed that factors (2) and (3), representing uncertainty associated with S parameter measurements, made a major contribution to overall uncertainty. Uncertainty values resulting from factors (2) and (3) were greater than those resulting from other factors. In addition, because the sensitivity coefficients of factors (2) and (3) were 2, compared to the sensitivity coefficient of 1 for other uncertainty factors, the influence of these two factors became even greater. These results indicate that it is vital to accurately measure the S parameters of the directional coupler for the purpose of reducing the quantity of uncertainty.

7 Conclusion

We described methods to calibrate high frequency power meters, which are used to measure 10 W, calibration systems, and methods to evaluate uncertainty. The simultaneous comparison method using a directional coupler enables the calibration of 50 Ω power meters in a frequency range between 100 kHz and 9 GHz, and the calibration of 75 Ω power meters in a frequency range between 1 MHz and 1 GHz. Maximum measurement capacities obtained from these calibrations were 1.9–3.1 % for 50 Ω power meters and 2.0–2.7 % for 75 Ω power meters. Previously, a maximum measurement capacity of 3.6 % had been recorded from the calibration of 50 Ω power meters performed in frequencies between 1 MHz and 2 GHz. Thus, the calibration system we developed achieved much better accuracy than previously developed systems. In this study, we evaluated uncertainty associated with the calibration of 75 Ω power meters for the first time.

We also determined factors that greatly affect calibration uncertainty. When the simultaneous comparison method was implemented using a directional coupler, it was vital to take accurate measurements of S parameters of the directional coupler. In the future, we will provide calibration service in a stable and accurate manner and aim to reduce the magnitude of calibration uncertainty by developing more accurate S parameter measurement techniques.

Table 4 Uncertainty budget

(a) 50 Ω power meter, 9 GHz

Uncertainty factor		Uncertainty	Distribution	Divisor	Standard uncertainty $u(x_i)$	Sensitivity coefficient $c(x_i)$	$ c(x_i) u(x_i)$
(1) STD (1mW)	0.0064	0.32%	Normal	1	0.0032	1	0.0032
(2) Directional coupler (S_{31})	0.1dB	0.58%	Normal	1	0.0058	2	0.0115
(3) Directional coupler (S_{21})	0.08dB	0.46%	Normal	1	0.0046	-2	0.0092
(4) Resolution of STD measurement	4 digits	0.05%	Uniform	$\sqrt{3}$	0.0003	-1	0.0003
(5) Resolution of DUT measurement	4 digits	0.05%	Uniform	$\sqrt{3}$	0.0003	1	0.0003
(6) Mismatch	0.9984	0.16%	U	$\sqrt{2}$	0.0011	1	0.0011
(7) Effect of heated ATT	0.001	0.10%	Uniform	$\sqrt{3}$	0.0006	1	0.0006
(8) Variability	0.0023	0.24%	Normal	$\sqrt{5}$	0.0011	1	0.0011
Relative standard uncertainty							0.0152
Relative expanded uncertainty ($k = 2$)							3.1%

(b) 75 Ω power meter, 1 GHz

Uncertainty factor		Uncertainty	Distribution	Divisor	Standard uncertainty $u(x_i)$	Sensitivity coefficient $c(x_i)$	$ c(x_i) u(x_i)$
(1) STD (1mW)	0.0046	0.23%	Normal	1	0.0023	1	0.0023
(2) Directional coupler (S_{31})	0.07dB	0.40%	Normal	1	0.0040	2	0.0081
(3) Directional coupler (S_{21})	0.04dB	0.23%	Normal	1	0.0023	-2	0.0046
(4) Resolution of STD measurement	4 digits	0.05%	Uniform	$\sqrt{3}$	0.0003	-1	0.0003
(5) Resolution of DUT measurement	4 digits	0.05%	Uniform	$\sqrt{3}$	0.0003	1	0.0003
(6) Mismatch	1.0063	0.63%	U	$\sqrt{2}$	0.0044	1	0.0044
(7) Effect of heated ATT	0.001	0.10%	Uniform	$\sqrt{3}$	0.0006	1	0.0006
(8) Variability	0.0013	0.14%	Normal	$\sqrt{4}$	0.0007	1	0.0007
Relative standard uncertainty							0.0106
Relative expanded uncertainty ($k = 2$)							2.2%

Appendix. A Derivation of Equation (4)

In the calibration system shown in Fig. 2, a signal source, DUT, and STD can be connected to ports #1, #2, and #3, respectively. This condition can be expressed using S parameters in the following equation:

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad (\text{A.1})$$

$$a_1 = a_G + \Gamma_G b_1 \quad (\text{A.2})$$

$$a_2 = \Gamma_{\text{ind}} b_2 \quad (\text{A.3})$$

$$a_3 = \Gamma_{\text{ins}} b_3 \quad (\text{A.4})$$

where the S matrix expresses the characteristics of the directional coupler (three port circuit), a_G is the waves coming out of the signal source (high frequency amplifier), Γ_G is the reflection coefficient of the signal source (high frequency amplifier), Γ_{ind} is the reflection coefficient of DUT, and Γ_{ins} is the reflection coefficient of STD. From these equations, the power P_{ind} , which enters the DUT, and

the power P_{inS} , which enters the STD, are each obtained by Equations (A.5) and (A.6).

$$P_{\text{inD}} = |b_2|^2 = \left| \frac{D_{(2S1)(11)}}{D} \right|^2 |a_G|^2 \quad (\text{A.5})$$

$$P_{\text{inS}} = |b_3|^2 = \left| \frac{D_{(3S1)(11)}}{D} \right|^2 |a_G|^2 \quad (\text{A.6})$$

D , $D_{(2S1)(11)}$, and $D_{(3S1)(11)}$ can be determined as follows.

$$D = \det \begin{bmatrix} 1 - S_{11}\Gamma_G & -S_{12}\Gamma_{\text{in}}^{\text{DUT}} & -S_{13}\Gamma_{\text{in}}^{\text{STD}} \\ -S_{21}\Gamma_G & 1 - S_{22}\Gamma_{\text{in}}^{\text{DUT}} & -S_{23}\Gamma_{\text{in}}^{\text{STD}} \\ -S_{31}\Gamma_G & -S_{32}\Gamma_{\text{in}}^{\text{DUT}} & 1 - S_{33}\Gamma_{\text{in}}^{\text{STD}} \end{bmatrix} \quad (\text{A.7})$$

$$D_{(2S1)(11)} = \det \begin{bmatrix} S_{21} & -S_{23}\Gamma_{\text{in}}^{\text{STD}} \\ S_{31} & 1 - S_{33}\Gamma_{\text{in}}^{\text{STD}} \end{bmatrix} = S_{21} \left\{ 1 - \left(S_{33} - S_{23} \frac{S_{31}}{S_{21}} \right) \Gamma_{\text{in}}^{\text{STD}} \right\} \quad (\text{A.8})$$

$$D_{(3S1)(11)} = \det \begin{bmatrix} 1 - S_{22}\Gamma_{\text{in}}^{\text{DUT}} & S_{21} \\ -S_{32}\Gamma_{\text{in}}^{\text{DUT}} & S_{31} \end{bmatrix} = S_{31} \left\{ 1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_{\text{in}}^{\text{DUT}} \right\} \quad (\text{A.9})$$

$\det[A]$ denotes the determinant of matrix A .

If two incident powers are measured simultaneously and their ratio is calculated, Equation (A.10) can be obtained using Equations (A.5) and (A.6).

$$\frac{P_M^{\text{DUT}}}{P_M^{\text{STD}}} = \frac{K_D}{K_S} \frac{P_{\text{inD}}}{P_{\text{inS}}} = \frac{K_D}{K_S} \frac{D_{(2S1)(11)}}{D_{(3S1)(11)}} = \frac{K_D}{K_S} \frac{|S_{21}|^2 \left| 1 - \left(S_{33} - S_{23} \frac{S_{31}}{S_{21}} \right) \Gamma_{\text{in}}^{\text{STD}} \right|^2}{\left| 1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_{\text{in}}^{\text{DUT}} \right|^2} \quad (\text{A.10})$$

Here, the relationships of

$P_{\text{inD}} = P_M^{\text{DUT}}/K_D$ and $P_{\text{inS}} = P_M^{\text{STD}}/K_S$ were used.

Equation (A.10) can be converted into Equation (A.11) below, which is virtually the same as Equation (4).

$$K_D = K_S \frac{|S_{31}|^2 \left(\frac{P_M^{\text{DUT}}}{P_M^{\text{STD}}} \right)}{\left| 1 - \left(S_{22} - S_{32} \frac{S_{21}}{S_{31}} \right) \Gamma_{\text{in}}^{\text{DUT}} \right|^2} \frac{1 - \left(S_{33} - S_{23} \frac{S_{31}}{S_{21}} \right) \Gamma_{\text{in}}^{\text{STD}}}{\left| 1 - \left(S_{33} - S_{23} \frac{S_{31}}{S_{21}} \right) \Gamma_{\text{in}}^{\text{STD}} \right|^2} \quad (\text{A.11})$$

During the process of deriving Equation (4), matrix D in Equation (A.7) is canceled out, so you do not need to calculate it. Matrix D contains the reflection coefficient of the signal source (high frequency amplifier) Γ_G . This coefficient can be canceled out if two incident powers are measured simultaneously and their ratio is calculated. In other words, it is unnecessary to determine the value of Γ_G . Thus, even if an adaptor is attached to port #1 in order to convert characteristic impedance from 50 to 75 Ω , this attachment does not affect calibration results. This is a

major advantage of the simultaneous comparison method.

[Appendix B] Derivation of Equation (7)

We will derive Equation (7), which combines uncertainties attributed to different sources, based on reference [11]. Equation (4), which determines calibration factor, reappears as Equation (B.1).

$$K_D = K_S \frac{|S_{31}|^2}{|S_{21}|^2} \left(\frac{P_M^{\text{DUT}}}{P_M^{\text{STD}}} \right) M \quad (\text{B.1})$$

There are six variables (input quantities x_i) in the equation. That is, uncertainties associated with these six variables affect calibration collectively. These uncertainties are expressed as $u(x_i)$. Also, sensitivity coefficient $c(x_i)$ can be determined by partially differentiating Equation (B.1). The uncertainty of DUT, $u(K_D)$, can be calculated using Equations (B.2) and (B.6).

$$u(K_D) = \sqrt{\sum_{i=1}^6 c(x_i)^2 u(x_i)^2} \quad (\text{B.2})$$

$$c(x_i) = \frac{\partial K_D}{\partial x_i} \quad (\text{B.3})$$

If all variables are plugged in, you can obtain Equation (B.4) below.

$$u(K_D) = \sqrt{\frac{c(K_S)^2 u(K_S)^2 + c(|S_{31}|)^2 u(|S_{31}|)^2 + c(|S_{21}|)^2 u(|S_{21}|)^2}{c(P_M^{\text{STD}})^2 u(P_M^{\text{STD}})^2 + c(P_M^{\text{DUT}})^2 u(P_M^{\text{DUT}})^2 + c(M)^2 u(M)^2}} \quad (\text{B.4})$$

$$c(K_S) = \frac{\partial K_D}{\partial K_S} = \frac{|S_{31}|^2}{|S_{21}|^2} \left(\frac{P_M^{\text{DUT}}}{P_M^{\text{STD}}} \right) M = \frac{K_D}{K_S} \quad (\text{B.5})$$

$$c(|S_{31}|) = \frac{\partial K_D}{\partial |S_{31}|} = 2K_S \frac{|S_{31}|}{|S_{21}|^2} \left(\frac{P_M^{\text{DUT}}}{P_M^{\text{STD}}} \right) M = 2 \frac{K_D}{|S_{31}|} \quad (\text{B.6})$$

$$c(|S_{21}|) = \frac{\partial K_D}{\partial |S_{21}|} = -2K_S \frac{|S_{31}|^2}{|S_{21}|^3} \left(\frac{P_M^{\text{DUT}}}{P_M^{\text{STD}}} \right) M = -2 \frac{K_D}{|S_{21}|} \quad (\text{B.7})$$

$$c(P_M^{\text{DUT}}) = \frac{\partial K_D}{\partial P_M^{\text{DUT}}} = K_S \frac{|S_{31}|^2}{|S_{21}|^2} \left(\frac{1}{P_M^{\text{STD}}} \right) M = \frac{K_D}{P_M^{\text{DUT}}} \quad (\text{B.8})$$

$$c(P_M^{\text{STD}}) = \frac{\partial K_D}{\partial P_M^{\text{STD}}} = K_S \frac{|S_{31}|^2}{|S_{21}|^2} \left(\frac{P_M^{\text{DUT}}}{P_M^{\text{STD}}} \right) \frac{1}{P_M^{\text{STD}}} M = \frac{K_D}{P_M^{\text{STD}}} \quad (\text{B.9})$$

$$c(M) = \frac{\partial K_D}{\partial M} = K_S \frac{|S_{31}|^2}{|S_{21}|^2} \left(\frac{P_M^{\text{DUT}}}{P_M^{\text{STD}}} \right) = \frac{K_D}{M} \quad (\text{B.10})$$

By plugging Equations (B.5) through (B.10), which are used to calculate sensitivity coefficients, into Equation (B.4), you can obtain Equation (B.11) below.

$$u(K_D) = \sqrt{\left(\frac{K_D}{K_S}\right)^2 u(K_S)^2 + \left(2 \cdot \frac{K_D}{|S_{31}|}\right)^2 u(|S_{31}|)^2 + \left(-2 \cdot \frac{K_D}{|S_{21}|}\right)^2 u(|S_{21}|)^2 + \left(\frac{K_D}{P_M^{STD}}\right)^2 u(P_M^{STD})^2 + \left(-1 \cdot \frac{K_D}{P_M^{DUT}}\right)^2 u(P_M^{DUT})^2 + \left(\frac{K_D}{M}\right)^2 u(M)^2}$$

(B.11)

Through the rearrangement of Equation (B.11), you can obtain Equation (B.12) below.

$$u(K_D) = K_D \sqrt{\left(\frac{1}{K_S}\right)^2 u(K_S)^2 + \left(\frac{2}{|S_{31}|}\right)^2 u(|S_{31}|)^2 + \left(\frac{-2}{|S_{21}|}\right)^2 u(|S_{21}|)^2 + \left(\frac{1}{P_M^{STD}}\right)^2 u(P_M^{STD})^2 + \left(\frac{-1}{P_M^{DUT}}\right)^2 u(P_M^{DUT})^2 + \left(\frac{1}{M}\right)^2 u(M)^2}$$

(B.12)

This equation determines uncertainty associated with the calibration factor of DUT. To express the uncertainty in terms of relative value, you can obtain Equation (B.13) by dividing both sides of Equation (B.12) by K_D and rearranging the equation. As a result, not only uncertainty associated with the calibration factor of DUT but also other types of uncertainties are converted into relative values. This conversion makes calculation easier.

$$\frac{u(K_D)}{K_D} = \sqrt{(+1)^2 \left\{ \frac{u(K_S)}{K_S} \right\}^2 + (+2)^2 \left\{ \frac{u(|S_{31}|)}{|S_{31}|} \right\}^2 + (-2)^2 \left\{ \frac{u(|S_{21}|)}{|S_{21}|} \right\}^2 + (+1)^2 \left\{ \frac{u(P_M^{STD})}{P_M^{STD}} \right\}^2 + (-1)^2 \left\{ \frac{u(P_M^{DUT})}{P_M^{DUT}} \right\}^2 + (+1)^2 \left\{ \frac{u(M)}{M} \right\}^2}$$

(B.13)

Then, by adding uncertainty due to variability among calibration results to Equation (B.12), you can obtain Equation (7), which combines seven types of uncertainties. Table 4 lists eight uncertainty factors, instead of seven. That is because the uncertainty of power measurements using a DUT was divided into uncertainty related to measurement resolution and uncertainty resulting from heated ATT.

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