2-4 Calibration of RF Attenuators

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The RF attenuator is one of devices to calibrate receivers such as spectrum analyzers to evaluate qualities of radio waves which are radiated from RF transmitters. The linearity, one of the receiver performance indices to show adequately the signal in accordance with the received power, can be evaluated using the RF attenuator, which can reduce the intensity of radio waves transmitting through a coaxial cable or a waveguide. The RF attenuator is also used to evaluate the RF output level of the signal generator. NICT performs calibration service of the RF attenuators as a device for calibration of receivers and signal generators and so on. This report describes the calibration method of RF attenuators of which operating frequency is from 10 MHz up to 18 GHz and the estimation method of measurement uncertainty.

1 Introduction

An RF attenuator is a device that can attenuate the radio waves that pass through by the desired amount. The National Institute of Information and Communications Technology (NICT) provides calibration values of RF attenuators, which are essential devices for validating "linearity" of display of received power (voltage), which is one of the parameters that shows receiving performance in a spectrum analyzer, etc. That is, as shown in Fig. 1, if one measures absolute values in an RF power meter, and changes attenuation of an RF attenuator, then one can validate whether the received power and voltage values displayed by the receiver maintain linearity and display correctly, and in the same circuit configuration, one can validate the attenuator in the signal generator.

This paper explains the definition of attenuation, calibration methods, and how to estimate calibration uncertainty, in the calibration service that NICT provides, especially for calibration of 10 MHz to 18 GHz RF attenuators registered in JCSS.

2 Definition of RF attenuation

2.1 Attenuation

As shown in Fig. 2, RF attenuation is the amount defined by (a) received power P_0 when the signal generator is directly connected to the receiver, divided by (b) received power P_1 when an attenuator is inserted, when the signal generator's reflection coefficient Γ_G and the receiver's reflection coefficient Γ_L are both 0 ($\Gamma_G = \Gamma_L = 0$) (here, $P_0 > P_1$). That is, RF attenuation is a relation of amounts of RF

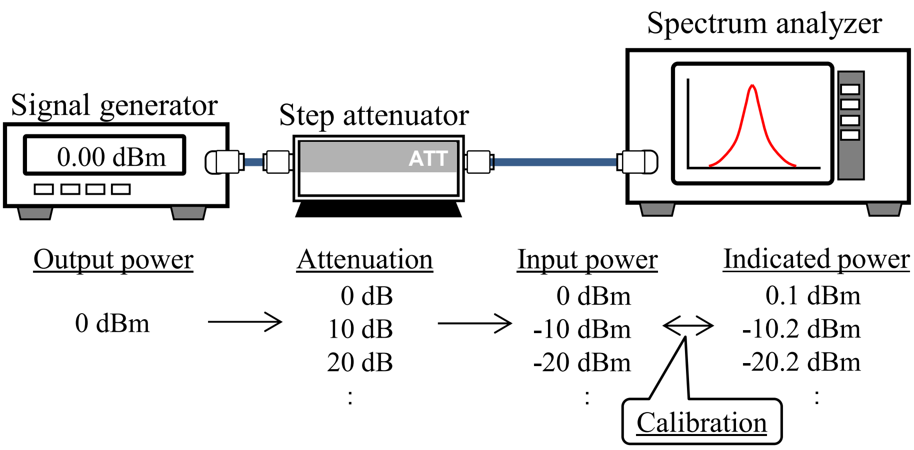

Fig. 1 Calibration of spectrum analyzer using an RF attenuator

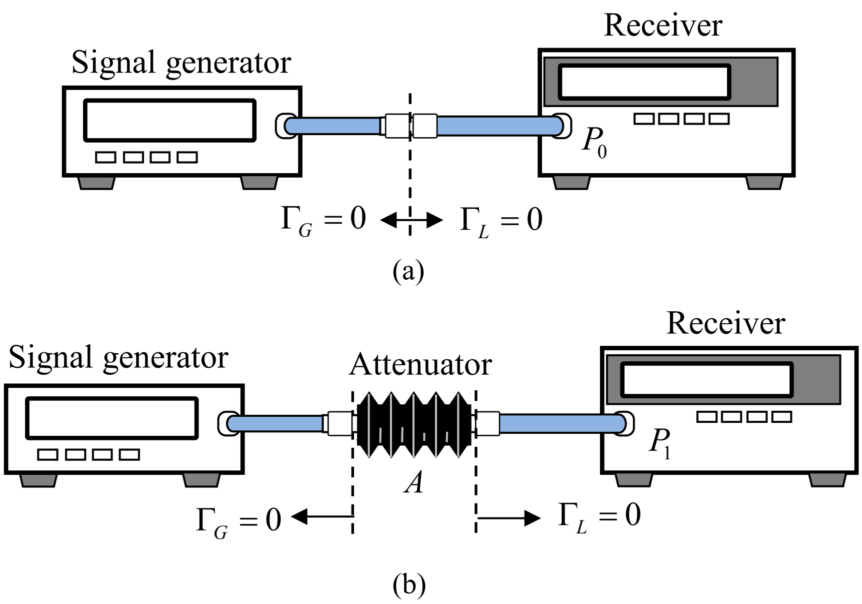

Fig. 2 Definition of RF attenuation

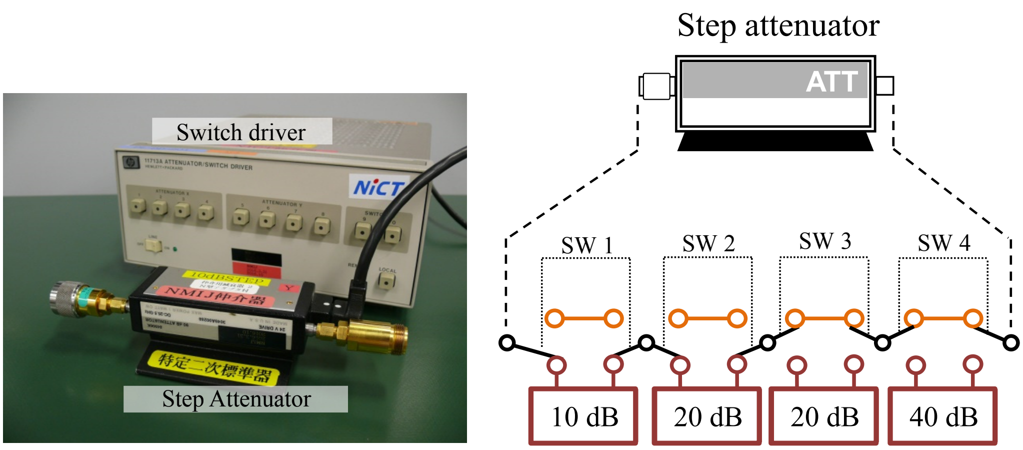

Fig. 3 Step attenuator: photo & internal structure

power; it is a dimensionless quantity defined by the following equation.

$$A = \left.\frac{P_0}{P_1}\right|_{\Gamma_G=\Gamma_L=0} \quad (1)$$

Also, it is usually expressed using dB.

$$A^{\mathrm{dB}} = 10\log_{10}\left.\frac{P_0}{P_1}\right|_{\Gamma_G=\Gamma_L=0} \quad [\mathrm{dB}] \quad (2)$$

Considering the case where one calibrates an attenuator and measures attenuation, the signal generator's reflection coefficient Γ_G and the receiver's reflection coefficient Γ_L are actually not 0, so one cannot obtain accurate values. As described later, this problem is treated as "uncertainty" associated with calibration results.

2.2 Incremental attenuation

The attenuator shown in Fig. 2 has a certain attenuation, so it is generally called a "fixed attenuator." In contrast, devices with variable attenuators are also sold in the market, called "variable attenuators"; these have the convenience of providing multiple attenuation amounts without the need to remove/attach/replace attenuators, so they are widely used. Figure 3 shows a sample photo and internal structure of an attenuator for which attenuation can be changed in steps, called a "step attenuator." It can change attenuation by switching relays, and is often equipped with attenuators that can change attenuation amount in 1 dB or 10 dB steps. Attenuators that can continuously change attenuation are also sold in the market.

As shown in Fig. 4, with a step attenuator connected between signal generator and receiver, we measure the

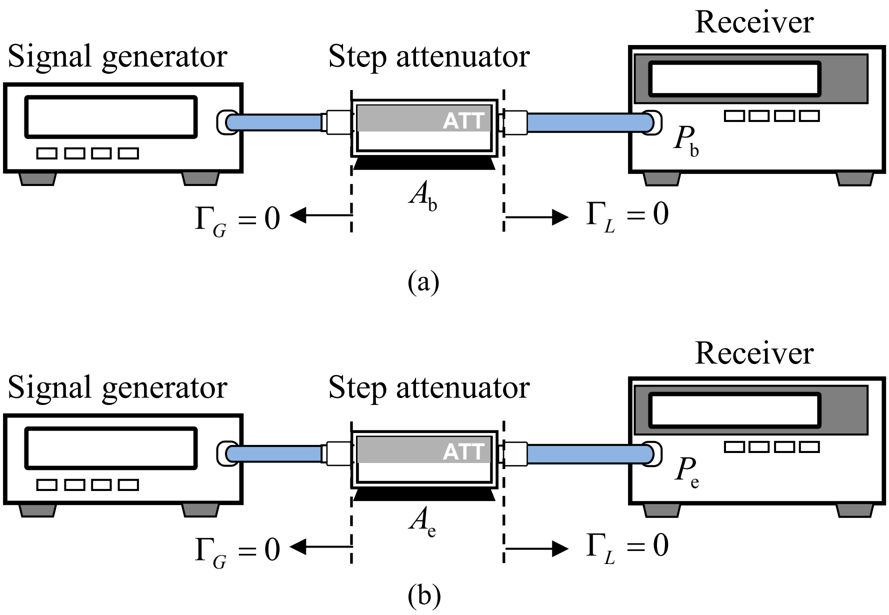

Fig. 4 Definition of incremental attenuation

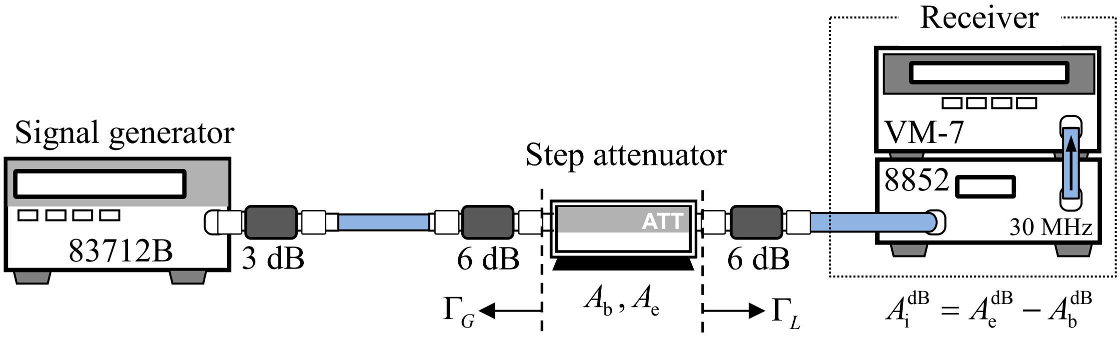

Fig. 5 RF attenuator calibration system

ratio of (a) reception power P_b before attenuation is changed (attenuation A_b), divided by (b) reception power P_e after attenuation is changed (attenuation A_e) (here, $A_e > A_b$). This attenuation ratio is provided by the following equation, which is called "incremental attenuation" to distinguish it from the usual attenuation.

$$A_i = \left.\frac{A_e}{A_b}\right|_{\Gamma_G=\Gamma_L=0} = \left.\frac{P_b}{P_e}\right|_{\Gamma_G=\Gamma_L=0} \quad (3)$$

Similar to attenuation, it is usually expressed using dB.

$$A_i^{dB} = 10\log_{10}\left.\frac{A_e}{A_b}\right|_{\Gamma_G=\Gamma_L=0} = 10\log_{10}\left.\frac{P_b}{P_e}\right|_{\Gamma_G=\Gamma_L=0} \quad [\text{dB}] \quad (4)$$

Incremental attenuation is simply called "attenuation" when confusion does not particularly occur. Also when measuring incremental attenuation of a step attenuator, the signal generator's reflection coefficient Γ_G and the receiver's reflection coefficient Γ_L are actually not 0, so one cannot obtain accurate values. As described later, this problem is treated as "uncertainty" associated with calibration results.

3 Calibration method

Here, we describe calibration of a step attenuator used when validating linearity of a receiver (determination of incremental attenuation), as shown in Fig. 1. Calibration of a step attenuator uses incremental attenuation of a secondary standard maintained and managed by NICT (hereinafter abbreviated as "STD"), and NICT calibrates the indicated values of an RF attenuation calibration system.

Figure 5 shows a calibration system. It uses a signal generator (Hewlett-Packard 83712B) and a receiving instrument (TEGAM VM-7 and 8852). The VM-7 is a receiver that contains an attenuator capable of precisely measuring signal strength only when the signal is at a frequency of 30 MHz. The 8852 is a frequency converter for converting the input signal to 30 MHz. To suppress reflection, fixed attenuators with 6 dB attenuation (Weinchel 44–6) are attached to the ends of the two coaxial cables that connect to the input/output terminals of the step attenuator.

10 MHz to 18 GHz frequency signals output from the signal generator pass through the step attenuator, which is between the fixed attenuators connected by coaxial cables. The signal that passes through is input into the receiving instrument. Specifically, the signal passes through a fixed attenuator with 3 dB attenuation (Weinchel 44–3) inserted to prevent saturation and for impedance matching, then it is input into the 8852 and converted to 30 MHz frequency, and the VM-7 measures the attenuation as a 30 MHz frequency signal. This method of converting frequency and using the change in signal strength of the intermediate frequency (now 30 MHz) to calibrate attenuation of an attenuator is called the "Intermediate Frequency Substitution Method"[1].

Validity of the attenuation value shown on the receiving instrument is confirmed by measuring a step attenuator called a STD, calibrated by a superior calibration organization. NICT ensures traceability to the national standard by the traceability system shown in Fig. 6. The STD is calibrated periodically by the national standard organization, so the accuracy of the calibration system is assured.

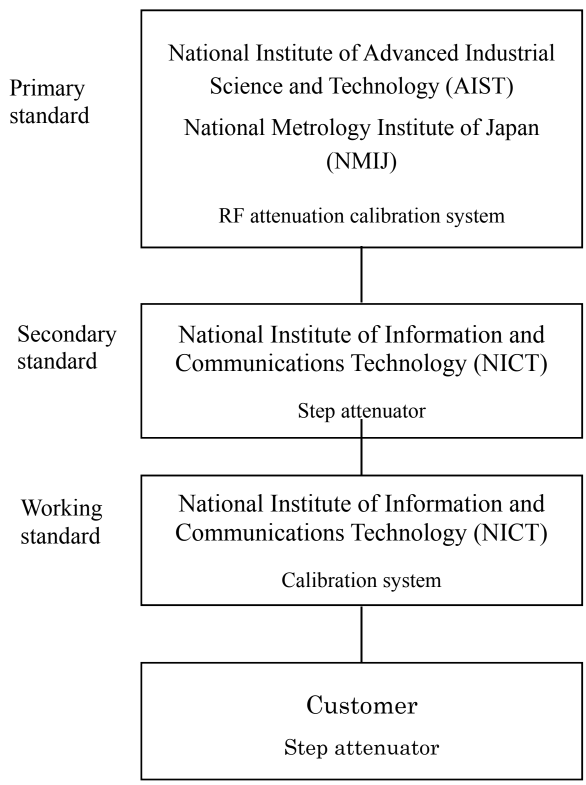

Fig. 6 Traceability chart

When the calibration system is confirmed as operating normally, then the calibration procedure is as follows[2].

(1) Connect the device being calibrated ("device under test (DUT)") to the coaxial cables to which the fixed attenuators are attached, with a 0 dB setting for the DUT (Step 1). At this time, read the A_b^{dB} value displayed on the receiving instrument.

(2) Set the DUT to the attenuation you want to calibrate (for example, 30 dB), and read the A_e^{dB} value displayed on the receiving instrument again.

(3) Use the following equation to determine incremental attenuation.

$$A_i^{dB} = A_e^{dB} - A_b^{dB} \quad [dB] \quad (5)$$

In the actual calibration work, operate the receiving instrument so the value measured in (1) is displayed as 0 dB, and arrange so the value measured and displayed in (2) becomes the incremental attenuation, then measure. Repeat the measurement five times to reduce the effects due to reproducibility of switching in the step attenuator.

4 Calibration results

Table 1 shows a list of calibration points that NICT

Table 1 Calibration range (10 MHz to 18 GHz)

(a)

Frequency	Attenuator	Attenuation (nominal value)
10 MHz, 30 MHz, 100 MHz, 500 MHz, 1 GHz, 5 GHz, 10 GHz	Step attenuator	10, 20, 30, 40, 50, 60, 70, 80, 90 dB
	Fixed attenuator	10, 20, 30, 40, 50, 60, 70 dB
18 GHz	Step attenuator	10, 20, 30, 40, 50, 60 dB
	Fixed attenuator	10, 20, 30, 40, 50, 60 dB
30 MHz	Step attenuator	1, 2, 3, 4, 5, 6, 7, 8, 9 dB

(b)

Frequency	Attenuator	Attenuation (nominal value)
Over 10 MHz to under 10 GHz	Step attenuator	10, 20, 30, 40, 50, 60, 70, 80, 90 dB
	Fixed attenuator	10, 20, 30, 40, 50, 60, 70 dB
Over 10 GHz to under 18 GHz	Step attenuator	10, 20, 30, 40, 50, 60 dB
	Fixed attenuator	10, 20, 30, 40, 50, 60 dB

Table 2 Results of NICT's validation of attenuation calibration system (for 30 dB, 60 dB, 90 dB)

Frequency	Attenuation (Nominal value)	Measured value by NICT	Calibrated value by NMIJ	Difference
10 MHz	30 dB	30.069 dB	30.084 dB	-0.015 dB
	60 dB	60.192 dB	60.186 dB	0.006 dB
	90 dB	89.970 dB	90.118 dB	-0.148 dB
30 MHz	30 dB	30.086 dB	30.082 dB	0.004 dB
	60 dB	60.189 dB	60.187 dB	0.002 dB
	90 dB	90.109 dB	90.114 dB	-0.005 dB
100 MHz	30 dB	30.085 dB	30.083 dB	0.002 dB
	60 dB	60.187 dB	60.185 dB	0.002 dB
	90 dB	90.095 dB	90.106 dB	-0.011 dB
500 MHz	30 dB	30.093 dB	30.083 dB	0.010 dB
	60 dB	60.198 dB	60.184 dB	0.014 dB
	90 dB	90.085 dB	90.099 dB	-0.014 dB
1 GHz	30 dB	30.082 dB	30.079 dB	0.003 dB
	60 dB	60.185 dB	60.173 dB	0.012 dB
	90 dB	90.060 dB	90.069 dB	-0.009 dB
5 GHz	30 dB	30.069 dB	30.073 dB	-0.004 dB
	60 dB	60.163 dB	60.175 dB	-0.012 dB
	90 dB	90.003 dB	90.056 dB	-0.053 dB
10 GHz	30 dB	30.064 dB	30.059 dB	0.005 dB
	60 dB	60.205 dB	60.187 dB	0.018 dB
	90 dB	90.079 dB	90.093 dB	-0.014 dB
18 GHz	30 dB	30.061 dB	30.100 dB	-0.039 dB
	60 dB	60.213 dB	60.259 dB	-0.046 dB

currently performs. The STDs (step attenuators) maintained and managed by NICT are calibrated by the National Metrology Institute of Japan (NMIJ). NICT performs calibration at the calibration points (frequencies and incremental attenuations) specified in calibration certificates issued by NMIJ shown in (a). Also, in tests of wireless equipment, radiated emission measurements, and antenna calibration, even at frequencies other than calibration points of (a), there are situations where we want to validate the receiver's performance, so as shown in (b), calibration values in the calibration certificate are interpolated in the frequencies' direction[3], and calibration points are expanded to enable calibration at any frequency, then JCSS registration is obtained and provided.

Table 2 shows differences between the values shown in our calibration system vs. calibration values by a superior calibration organization, for 30, 60 and 90 dB attenuation. Figure 7 shows a graph of these differences. Looking at these results, first, compared to 30 dB and 60 dB, we see

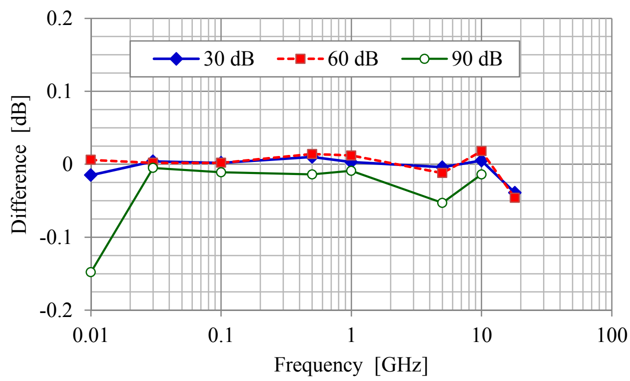

Fig. 7 Results of NICT attenuation calibration system validated using a secondary standard

that at 90 dB has larger differences between our measured values and the superior calibration. We think this may be caused because the frequency conversion part of the receiving instrument has insufficient linearity. Also, at 10 MHz it reaches −0.148 dB, which could be due to leaking signal of the reference signal source (10 MHz) in the receiving instrument. Next, we see that differences at 18 GHz frequency tend to be greater at 10 GHz or less. This is due to mismatch arising because the calibration system and attenuator have reflection. Actually, section **5.1** (4) shows results of seeking the effects due to mismatch; we confirmed that as frequency rises, the effects of mismatch increase.

5 Uncertainty

Calibration results have uncertainty. The size of uncertainty (unit is dB) that occurs due to multiple factors as described below can be estimated by using the following equation.

$$u\left(A_i^{dB}\right) = \sqrt{\sum_{n=1}^{N} \left\{\left|c_n\right| \cdot u\left(x_n\right)\right\}^2} \quad \text{[dB]} \quad (6)$$

Here,

$u\left(A_i^{dB}\right)$: Standard uncertainty of DUT

$\left|c_n\right|$: Absolute value of the sensitivity coefficient that shows the degree that the nth uncertainty factor contributes to $u\left(A_i^{dB}\right)$ (all are 1 in this calibration)

$u\left(x_n^{dB}\right)$: Standard uncertainty of uncertainty factor of item n

N : Number of uncertainty items (10 items)

This result is called standard uncertainty. Usually, uncertainty is expressed as expanded uncertainty estimated at a level of confidence of 95 %. Thus, the expanded uncertainty is obtained by multiplying uncertainty by the coverage factor k = 1.96. It should be noted that k = 2 is widely used for convenience.

Items that cause uncertainty are explained below. Causes of uncertainty are divided into "Uncertainty when calibrating a calibration system, using a step converter calibrated as a standard device in the superior calibration organization (NMIJ)" described in section **5.1**, and "Uncertainty when calibrating a device under test, using a calibration system" described in section **5.2**.

5.1 Uncertainty when calibrating a calibration system

As shown in Fig. 6, standard devices (step attenuators) maintained and managed by NICT are calibrated by the National Metrology Institute of Japan (NMIJ). NICT calibrates calibration systems by reference to calibration values and uncertainties specified in calibration certificates issued by NMIJ. At this time, the seven uncertainty causes described below add uncertainty to calibration results of the calibration system.

(1) Uncertainty of Calibration of Standard Device in Superior Calibration Organization

This is the value of uncertainty documented in a calibration certificate issued from NMIJ which is a superior calibration organization. NMIJ also calibrates step attenuators, so uncertainty exists in the calibration results.

The calibration certificate states that the value of uncertainty provided in the certificate is equal to expanded uncertainty at a level of confidence of approximately 95 % when coverage factor k = 2. Therefore, standard uncertainty is the value of uncertainty provided in the calibration certificate divided by coverage factor k = 2.

(2) Resolution of a Digital Indication

The value displayed in the receiving instrument has a 0.001 dB display resolution. Therefore, these are uncertain values where one cannot know values at a level of precision greater than 0.001 dB. For example, when 30.004 dB displays, as shown in Fig. 8, the true value of attenuation could be in the range from 30.0035 dB to 30.0045 dB, that is, in the range 30.004 dB ± 0.0005 dB. The possibility that the true value exists is uniform across that range. Here, we estimate that the digital display resolution has an uncertainty of 0.0029 dB (= 0.005 / $\sqrt{3}$ dB) as standard uncertainty.

(3) Variability of Measurements (20 Times)

When using a STD to calibrate a calibration system, measure 20 times and validate. By measuring 20 times and using the average value as the calibration result, one reduces variability of the attenuation that occurs when the switch is switched in the STD. The average value of 20 measurements is used as the calibration result, so the uncertainty value is obtained by dividing the experimental standard deviation of 20 measurements by the

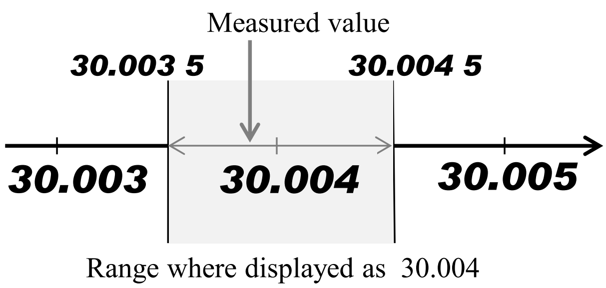

Fig. 8 Uncertainty of resolution of a digital indication

square root of 20. The standard deviation of the average value is the same as experimental standard deviation divided by the square root of the number of measurements[4].

(4) Mismatch

The reflection coefficients of the signal generator and receiving instrument of the calibration system are not $\Gamma_G=\Gamma_L=0$, so the bias of calibration results that occurs due to mismatch vs. the STD must be recorded as uncertainty. Equation (A.12) is an expression by S-parameters of incremental attenuation; in cases where $\Gamma_G=\Gamma_L=0$ is not true, it becomes Equation (A.11), so one can say that the difference between the two equations is the effect of mismatch. Now, if we substitute Equation (A.12) with Equation (A.11), we get

$$L_{\mathrm{i}}^{\mathrm{dB}} = A_{\mathrm{i}}^{\mathrm{dB}} - 10\log_{10}\frac{\left|(1-S_{11\mathrm{b}}\Gamma_G)(1-S_{22\mathrm{b}}\Gamma_L)-S_{21\mathrm{b}}S_{12\mathrm{b}}\Gamma_G\Gamma_L\right|^2}{\left|(1-S_{11\mathrm{e}}\Gamma_G)(1-S_{22\mathrm{e}}\Gamma_L)-S_{21\mathrm{e}}S_{12\mathrm{e}}\Gamma_G\Gamma_L\right|^2}\ [\mathrm{dB}] \tag{7}$$

Here, if we interchange denominator and numerator of the second term on the right, and reverse the sign, we define

$$M_{\mathrm{i}}^{\mathrm{dB}} = +10\log_{10}\frac{\left|(1-S_{11\mathrm{e}}\Gamma_G)(1-S_{22\mathrm{e}}\Gamma_L)-S_{21\mathrm{e}}S_{12\mathrm{e}}\Gamma_G\Gamma_L\right|^2}{\left|(1-S_{11\mathrm{b}}\Gamma_G)(1-S_{22\mathrm{b}}\Gamma_L)-S_{21\mathrm{b}}S_{12\mathrm{b}}\Gamma_G\Gamma_L\right|^2}\ [\mathrm{dB}] \tag{8}$$

and consider uncertainty. First, use natural logarithm (ln) to express Equation (8).

$$\begin{aligned} M_{\mathrm{i}}^{\mathrm{dB}} &= \frac{20}{\ln 10}\ln\left|\frac{(1-S_{11\mathrm{e}}\Gamma_G)(1-S_{22\mathrm{e}}\Gamma_L)-S_{21\mathrm{e}}S_{12\mathrm{e}}\Gamma_G\Gamma_L}{(1-S_{11\mathrm{b}}\Gamma_G)(1-S_{22\mathrm{b}}\Gamma_L)-S_{21\mathrm{b}}S_{12\mathrm{b}}\Gamma_G\Gamma_L}\right| \\ &= 8.686\cdot\ln\left|\frac{(1-S_{11\mathrm{e}}\Gamma_G)(1-S_{22\mathrm{e}}\Gamma_L)-S_{21\mathrm{e}}S_{12\mathrm{e}}\Gamma_G\Gamma_L}{(1-S_{11\mathrm{b}}\Gamma_G)(1-S_{22\mathrm{b}}\Gamma_L)-S_{21\mathrm{b}}S_{12\mathrm{b}}\Gamma_G\Gamma_L}\right|\ [\mathrm{dB}] \end{aligned} \tag{9}$$

Here, when x is sufficiently smaller than 1, using the fact that the approximation $\ln(1+x)\approx x$ is true, we can express this as

$$M_{\mathrm{i}}^{\mathrm{dB}} \approx 8.686\cdot\left\{|\Gamma_G|^2|S_{11\mathrm{b}}-S_{11\mathrm{e}}|^2+|\Gamma_L|^2|S_{22\mathrm{b}}-S_{22\mathrm{e}}|^2+|\Gamma_G|^2|\Gamma_L|^2|S_{21\mathrm{b}}^2-S_{21\mathrm{e}}^2|^2\right\}\ [\mathrm{dB}] \tag{10}$$

This approximation formula requires not only on amplitude but also phase information for S-parameters of the STD (step attenuator), but it only requires amplitude information for reflection coefficients of the signal generator and receiving instrument. Measurement of the reflection coefficient of the signal generator is generally difficult, so Equation (10) is used widely as an approximation formula to evaluate mismatch uncertainty[1][5]. For the probability density distribution of uncertainty, the phase of reflection coefficient of the signal generator and receiving instrument is unclear, so it has a U-shaped distribution taking any value from 0° to 360°.

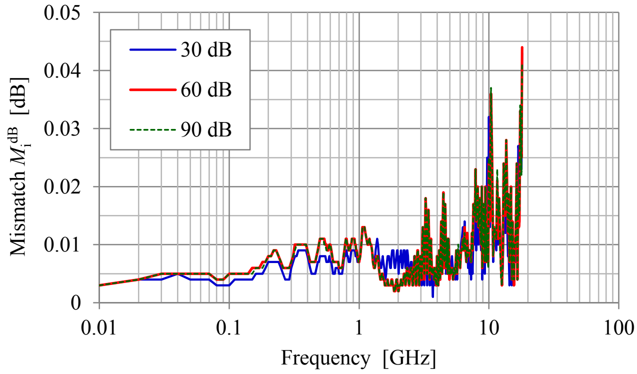

Fig. 9 Mismatch effects

Figure 9 shows the results of obtaining the effects due to mismatch between the STD and the calibration system. A vector network analyzer was used to measure the actual reflection coefficients. The signal generator's reflection coefficients differ when signal is being output (ON) vs. when not being output (OFF), so while signals are being output at 20 GHz frequency, the 10 MHz to 18 GHz reflection coefficients are measured and used in the calculation. Figure 9 indicates that the values are almost the same when 60 dB and 90 dB are compared. This is because attenuation is large, so the attenuator's reflection coefficients converge on a certain value. Also, there are large effects from near 10 GHz frequency, reaching a maximum of approximately 0.05 dB. This result matches the difference (–0.046 dB) that occurs at 18 GHz in Table 2.

(5) Changes Due to Differences vs. Measurement Environment of the Superior Calibration Organization

For the step attenuator used as the STD, if there are changes in the temperature of the environment used, then resistance values of the internal resistance parts can change and attenuation can vary. It is documented that calibration of the STD is to be performed in a room with its temperature controlled in the range 23 °C ± 1 °C, but the calibration room where NICT's calibrations are performed are controlled at 23 °C ± 2 °C. Therefore, calibration is performed under conditions that are less strict than the conditions the STD was calibrated under, so the degree of its effects is viewed as an item that creates uncertainty. To clarify this effect, we actually made measurements at different temperatures; results indicated that if in the range 23 °C ± 2 °C, then the effects due to differences in temperature were sufficiently small, lower than the variability of measurements.

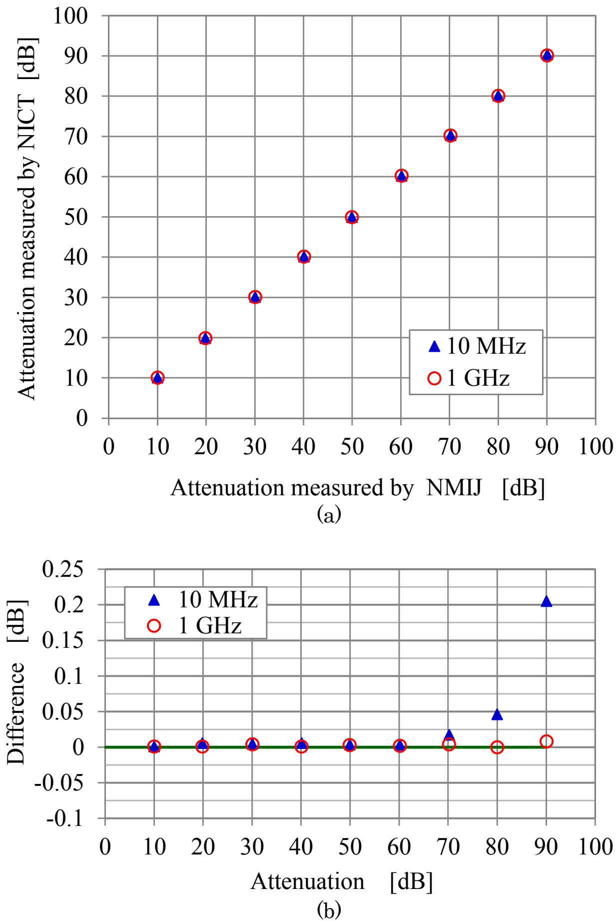

Fig. 10 Linearity of receiving instrument

(6) Changes over Time

We validate the degree that the incremental attenuation of the STD changes with time. We conducted a study using results from calibration system during the past nine years. We did not find certain expected trends such as a gradual increase in attenuation, but we decided to record the maximum value of change amount in 1 year as uncertainty. Assuming that the chance of the true value existing is uniform across this range, the probability distribution is treated as a uniform distribution.

(7) Linearity of receiving instrument

We verify linearity of a receiving instrument by using an STD and comparing values measured by the receiving instrument with the values specified in calibration certification of STD. Figure 10 shows an example of linearity at 10 MHz and 1 GHz. In Figure 10 (a), the vertical axis is the measurement value of incremental attenuation by the calibration system, and the horizontal axis is the calibration value at the superior calibration organization (value specified in the calibration certification); the ▲ (blue) are 10 MHz, and the ○ (red) are 1 GHz results. Figure 10 (b) shows differences between measured points and the regression line. The maximum differences obtained at nine points from 10 dB to 90 dB are recorded as causes of uncertainties. At 1 GHz, the difference is a maximum 0.014 dB when incremental attenuation is 90 dB, so 0.014 dB is recorded as a cause of uncertainty. On the other hand, at 10 MHz, differences of measured values rapidly increase at 80 dB and 90 dB. Therefore, at 10 MHz frequency, looking at performance of uncertainty evaluations for 10 dB to 70 dB separately from those for 80 dB and 90 dB, the maximum difference from the regression line obtained from seven points 10 dB to 70 dB, and the differences between measured values and calibrated values from the superior calibration organization at 80 dB and 90 dB, are each recorded as uncertainty. The trend at 30 MHz was similar to that at 10 MHz, so we assessed uncertainty for 10 dB to 70 dB separately from uncertainty for 80 dB and 90 dB.

Differences from the regression line can occur because of the saturation of the frequency conversion device, internal noise and signal leaks; the numbers of this uncertainty can include the effects of noise and leaks. One may think that noise and leaks are usually recorded separately as causes of uncertainty of the receiving instrument, but they can be recorded in duplicate, so we do not record them separately.

Deviations from this regression line could be "systematic bias" that always occurs, so there is no probability density distribution. Therefore, we record the same values obtained as standard uncertainty[6].

5.2 Uncertainty when a device under test is calibrated

Uncertainty can be caused by the following three items when calibrating a step attenuator of the DUT using a calibration system that was calibrated using an STD.

(8) Resolution of a Digital Indication

Same as (2)

(9) Repeatability of Measurements (5 times)

Same as (3). However, measurements are taken 5 times, so experimental standard deviation divided by the square root of 5 is recorded as the standard uncertainty of the average value.

(10) Mismatch

Same as (4). A vector network analyzer is used to measure the S-parameters of the DUT, Equation (10) is used to obtain the uncertainty, and it is recorded.

5.3 Uncertainty of frequency extension

this refers to uncertainty occurring when determining the calibration value at any frequency after the calibration values of the STD were interpolated at the calibration points (frequencies and attenuation) shown in Table 1 (a). When interpolating and determining the calibration value, use the combinations of calibration points shown in Table 3, and use the regression line to determine calibration results[7]. Uncertainty may occur when one determines calibration values at any frequency using the regression line. As described in reference [7], such uncertainty should be combined with the uncertainty occurring during the calibration of the STD in a superior organization described in section **5.1** (1). Then the combined uncertainty should be added to the budget table as standard uncertainty. Table 4 shows the amount of increase in standard uncertainty.

Table 3 Calibration points used when extending frequencies

Extended Frequency Range	Attenuation	Frequencies of calibration values by NMIJ using interpolation
Over 10 MHz to under 1 GHz	10, 20, 30, 40, 50, 60, 70, 80, 90 dB	10 MHz, 30 MHz, 100 MHz, 500 MHz, 1 GHz(5 points)
Over 1 GHz to under 10 GHz		1 GHz, 5 GHz, 10 GHz(3 points)
Over 10 GHz to under 18 GHz	10, 20, 30, 40, 50, 60 dB	5 GHz, 10 GHz, 18 GHz(3 points)

Table 4 Increase in uncertainty due to frequency ranges extended by interpolation

Extended Frequency Range	Attenuation	Expanded Uncertainty by NMIJ	Extended Uncertainty for Frequency range extension
Over 10 MHz to under 1 GHz	10, 20, 30, 40, 50, 60, 70, 80, 90 dB	0.020 dB	0.030 dB
Over 1 GHz to under 10 GHz		0.020 dB	0.044 dB
Over 10 GHz to under 18 GHz	10, 20, 30, 40, 50, 60 dB	0.010 dB (18 GHz)	0.042 dB

5.4 Calibration and measurement capability

Table 5 shows an example of a budget table that summarizes the causes of uncertainty described above. Table 5 has the calibration and measurement capability ("CMC") when calibrating 10 dB to 90 dB incremental attenuation, at 10 MHz to 10 GHz of the step attenuator. However, a different budget table is prepared for 80 dB and 90 dB of 10 MHz and 30 MHz, where linearity of the receiving instrument deteriorates. The uncertainty is the amount assigned to each calibration point, but to handle it simply, in the frequency range mentioned above, the largest value is taken for each, and the calculation is performed.

The budget table is divided into two parts: uncertainty of measurements when using an STD to validate the calibration system, and uncertainty when measuring the attenuator of a DUT.

CMC is obtained assuming no mismatch of the DUT, and no variability of measurements. That is, it is determined by substituting 0 for (10) mismatch and (9) measurement variability. In actual calibrations, numbers are combined by entering them in these fields.

Table 5 shows that expanded uncertainty accompanying calibration results are at least 0.087 dB, when calibrated at 10 MHz to 10 GHz frequency at 10 dB to 90 dB incremental attenuation (excluding 10 MHz & 30 MHz at 80 dB & 90 dB). Looking at Table 5, we can see the causes that increase uncertainty. That is, from Table 5, we see that uncertainty is increased by mismatch that occurs due to reflection of the calibration system, and by linearity. In the future, to provide reliable calibration results with small uncertainty, it would be effective to improve mismatch and linearity of the calibration system. Table 6 is the CMC in the calibration ranges shown in Table 1.

6 Conclusion

We explained the calibration techniques that NICT uses, and the way of thinking about uncertainty associated with calibration results, for an RF attenuator needed to validate performance and calibrate a receiver and signal generator used to test wireless equipment. By using the uncertainty budget table, and closely investigating causes of greater uncertainty, we showed that reflection in the calibration system and linearity of the receiving instrument are causes that increase uncertainty. To solve this problem, calibration precision could be improved by using an impedance tuner to reduce reflection instead of a fixed attenuator, and by preparing a receiving instrument with

Table 5 Uncertainty budget

10 MHz, 30 MHz, 100 MHz, 500 MHz, 1 GHz, 5 GHz, 10 GHz Uncertainty of calibration results of 10, 20, 30, 40, 50, 60, 70, 80, 90 dB step attenuator (excluding 10 MHz & 30 MHz at 80 dB & 90 dB)			
Sources of uncertainty	dB	Distribution	Contribution dB
Uncertainty occurring when calibrating a calibration system using STD			
(1) Uncertainty of calibration results by NMIJ	0.020 (k=2)	Normal	0.010
(2) Resolution of a Digital Indication	0.0005	Rectangle	0.0003
(3) Repeatability (standard deviation of 20 times measurement)	0.0008	Normal	0.0008
(4) Mismatch	0.033	U-shape	0.0233
(5) Change due to difference from NMIJ's environment (temperature) of STD	0.000	Rectangle	0.000
(6) Change over time	0.009	Rectangle	0.0052
(7) Linearity	0.0196	–	0.0196
Uncertainty of calibration system u_s			0.0325
Uncertainty when using a calibration system to measure an DUT			
(8) Resolution of a Digital Indication	0.0005	Rectangle	0.0003
(9) Repeatability (standard deviation of 5 times measurement)	Measured	Normal	0
(10) Mismatch	Measured	U-shape	0
Uncertainty of DUT measurement u_D			0.0003
Combined Standard Uncertainty u_c			0.0325 dB
Expanded Uncertainty (level of Confidence of approx. 95 %)			0.065 dB

excellent linearity.

Recent years have brought dramatic improvements in performance of spectrum analyzers and other receivers, especially progress in higher frequencies, and broader dynamic range of receiving levels. RF attenuators are essential reference devices for validating performance of receivers, and future issues are provision of attenuation standards in millimeter wave bands and Tera hertz waves, and provision of standards for large attenuators greater than 100 dB.

Appendix: Relationship between attenuation, Insertion loss, Substitution loss, and S-parameters

"Insertion loss" and "S-parameter (S_{21})" are amounts similar to attenuation. These differences are described clearly below[1]. Figure A.1 shows when an attenuator is inserted between a signal generator and receiver to measure attenuation, using S-parameters to express it, with the attenuator's input terminal as Port 1, and the output terminal and Port 2.

In this case,

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{A.1}$$

$$a_1 = a_G + b_1 \Gamma_G \tag{A.2}$$

$$a_2 = b_2 \Gamma_L \tag{A.3}$$

Here, Γ_G is the reflection coefficient of the signal generator, and Γ_L is the reflection coefficient of the receiver. Using Equations (A.1) to (A.3) to obtain incident power into the receiver, we obtain

$$P_2 = |b_2|^2 = \frac{|S_{21}|^2}{|(1 - S_{11}\Gamma_G)(1 - S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_G\Gamma_L|^2} |a_G|^2 \tag{A.4}$$

When we remove the attenuator, and the signal generator is directly connected to the receiver, the incident power is

Table 6 Calibration and measurement capability (CMC)

(a)

<table>
<tr><th>Frequency</th><th>Attenuator</th><th>Attenuation
(nominal value)</th><th colspan="3">Expanded uncertainty
(level of confidence of approx. 95 %)</th></tr>
<tr><td rowspan="6">10 MHz, 30 MHz,
100 MHz,
500 MHz,
1 GHz, 5 GHz,
10 GHz</td><td rowspan="5">Step attenuator</td><td rowspan="5">10, 20, 30, 40, 50, 60, 70, 80, 90 dB</td><td colspan="3">0.065 dB</td></tr>
<tr><td rowspan="2">10 MHz</td><td>80 dB</td><td>0.16 dB</td></tr>
<tr><td>90 dB</td><td>0.48 dB</td></tr>
<tr><td rowspan="2">30 MHz</td><td>80 dB</td><td>0.083 dB</td></tr>
<tr><td>90 dB</td><td>0.19 dB</td></tr>
<tr><td>Fixed attenuator</td><td>10, 20, 30, 40, 50, 60, 70 dB</td><td colspan="3">0.066 dB</td></tr>
<tr><td rowspan="2">18 GHz</td><td>Step attenuator</td><td>10, 20, 30, 40, 50, 60 dB</td><td colspan="3">0.13 dB</td></tr>
<tr><td>Fixed attenuator</td><td>10, 20, 30, 40, 50, 60 dB</td><td colspan="3">0.13 dB</td></tr>
<tr><td>30 MHz</td><td>Step attenuator</td><td>1, 2, 3, 4, 5, 6, 7, 8, 9 dB</td><td colspan="3">0.018 dB</td></tr>
</table>

(b)

<table>
<tr><th>Frequency</th><th>Attenuator</th><th>Attenuation
(nominal value)</th><th colspan="3">Expanded uncertainty
(level of confidence of approx. 95 %)</th></tr>
<tr><td rowspan="6">Over 10 MHz
to under 1 GHz</td><td rowspan="5">Step attenuator</td><td rowspan="5">10, 20, 30, 40, 50, 60, 70, 80, 90 dB</td><td colspan="3">0.069 dB</td></tr>
<tr><td rowspan="2">Over 10 MHz to under 30 MHz</td><td>80 dB</td><td>0.17 dB</td></tr>
<tr><td>90 dB</td><td>0.48 dB</td></tr>
<tr><td rowspan="2">Over 30 MHz to under 100 MHz</td><td>80 dB</td><td>0.086 dB</td></tr>
<tr><td>90 dB</td><td>0.19 dB</td></tr>
<tr><td>Fixed attenuator</td><td>10, 20, 30, 40, 50, 60, 70 dB</td><td colspan="3">0.069 dB</td></tr>
<tr><td rowspan="2">Over 1 GHz
to under 10 GHz</td><td>Step attenuator</td><td>10, 20, 30, 40, 50, 60, 70, 80, 90 dB</td><td colspan="3">0.076 dB</td></tr>
<tr><td>Fixed attenuator</td><td>10, 20, 30, 40, 50, 60, 70 dB</td><td colspan="3">0.077 dB</td></tr>
<tr><td rowspan="2">Over 10 GHz
To under 18 GHz</td><td>Step attenuator</td><td>10, 20, 30, 40, 50, 60 dB</td><td colspan="3">0.13 dB</td></tr>
<tr><td>Fixed attenuator</td><td>10, 20, 30, 40, 50, 60 dB</td><td colspan="3">0.13 dB</td></tr>
</table>

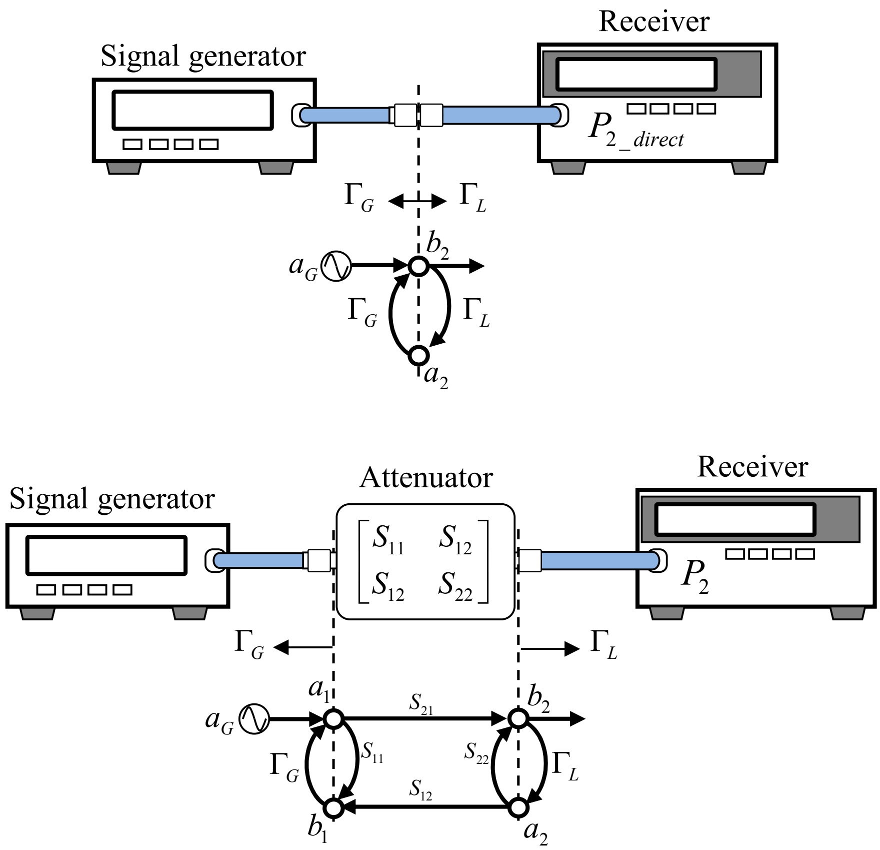

Fig. A.1 Expression using S-parameters of attenuation measurements

substituted with the following matrix equation.

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{A.5}$$

Then we obtain the following equation.

$$P_{2\_direct} = \frac{1}{\left|1-\Gamma_G\Gamma_L\right|^2}\left|a_G\right|^2 \tag{A.6}$$

Now, L is the ratio when directly connected vs. when the attenuator is connected. L is the amount called "Insertion Loss". From Equations (A.4) to (A.6), L is

$$L = \frac{P_{2\_direct}}{P_2} = \frac{1}{\left|S_{21}\right|^2}\frac{\left|(1-S_{11}\Gamma_G)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_G\Gamma_L\right|^2}{\left|1-\Gamma_G\Gamma_L\right|^2} \tag{A.7}$$

Expressed as dB, it becomes

$$L^{dB} = 10\log_{10}\frac{\left|(1-S_{11}\Gamma_G)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_G\Gamma_L\right|^2}{\left|S_{21}\right|^2\left|1-\Gamma_G\Gamma_L\right|^2} \quad \text{[dB]} \tag{A.8}$$

From Equations (A.7) and (A.8), we see that insertion loss is a value that changes due to reflection coefficients of the signal generator and receiver; this amount does not show a unique characteristic of the attenuator. Therefore, the definition of attenuation was the value when $\Gamma_G = \Gamma_L = 0$, so consider substituting $\Gamma_G=\Gamma_L=0$ in Equation (A.8).

In this case, we obtain

$$A^{\mathrm{dB}} = L^{\mathrm{dB}}\Big|_{\Gamma_G=\Gamma_L=0} = -10\log_{10}\left|S_{21}\right|^2 \quad \text{[dB]} \tag{A.9}$$

From the above, we can say the following about attenuation.

- Attenuation is the value of insertion loss when the reflection coefficients of the signal generator and the receiver are 0.
- Attenuation is the inverse of the square of the amplitude of S-parameters of the attenuator. When expressed as dB, it is the same as the amount of the negative value of the square of the amplitude of S_{21}.
- Attenuation is a unique value of a fixed attenuator; it does not depend on qualities (reflection coefficients) of the signal generator and the receiver.

Even when using a step attenuator to measure, one can think using S-parameters in the same way. Now, if S-parameters in the state before attenuation changes are $\{S_{11\,b}, S_{21\,b}, S_{12\,b}, S_{22\,b}\}$, and if S-parameters after attenuation changes are $\{S_{11\,e}, S_{21\,e}, S_{12\,e}, S_{22\,e}\}$, then use Equation (A.7) for the insertion loss ratio

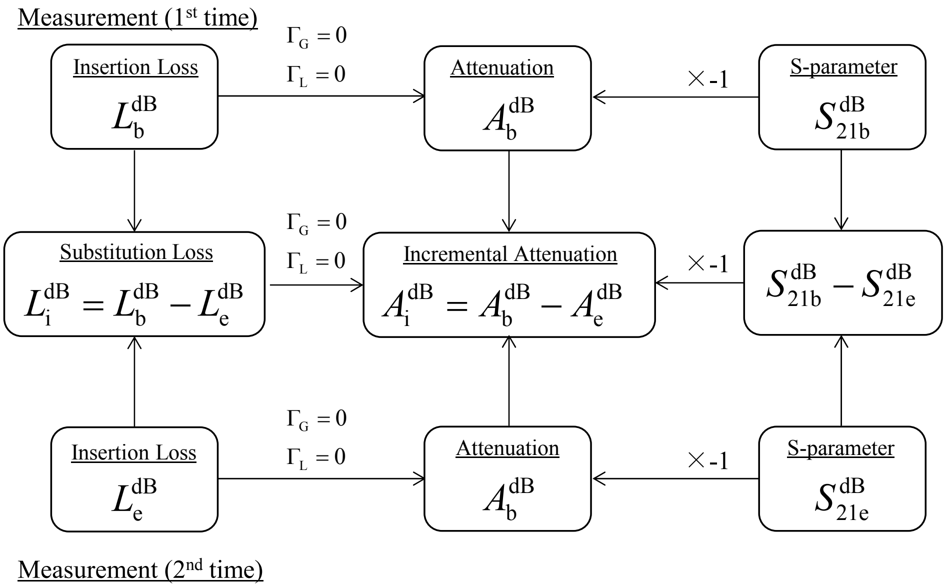

Fig. A.2 Relationship between attenuation, insertion loss, substitution loss, and S-parameters

$$L_i = \frac{|S_{21b}|^2}{|S_{21e}|^2} \frac{|(1-S_{11e}\Gamma_G)(1-S_{22e}\Gamma_L)-S_{21e}S_{12e}\Gamma_G\Gamma_L|^2}{|(1-S_{11b}\Gamma_G)(1-S_{22b}\Gamma_L)-S_{21b}S_{12b}\Gamma_G\Gamma_L|^2} \quad (A.10)$$

If expressed as dB, we obtain

$$L_i^{dB} = -10\log_{10}\frac{|S_{21e}|^2}{|S_{21b}|^2}\frac{|(1-S_{11b}\Gamma_G)(1-S_{22b}\Gamma_L)-S_{21b}S_{12b}\Gamma_G\Gamma_L|^2}{|(1-S_{11e}\Gamma_G)(1-S_{22e}\Gamma_L)-S_{21e}S_{12e}\Gamma_G\Gamma_L|^2} \text{ [dB]} \quad (A.11)$$

This value is called "Substitution Loss". Moreover, if we substitute Γ_G=Γ_L=0 in this equation, we obtain

$$A_i^{dB} = L_i^{dB}\Big|_{\Gamma_G=\Gamma_L=0} = -10\log_{10}\frac{|S_{21e}|^2}{|S_{21b}|^2} \quad \text{[dB]} \quad (A.12)$$

Even in the case of "incremental attenuation", like attenuation, we can say the following.

- Incremental attenuation is the value of substitution loss when the reflection coefficients of the signal generator and the receiver are 0.
- Incremental attenuation is the ratio of the inverse of the square of the amplitude of the S-parameters of the attenuator; when expressed in dB, it is the same as the amount calculated by taking the ratio of the square value of the amplitude of S_{21} and assigning a negative sign to it.
- Incremental attenuation is a unique value of the step attenuator, not dependent on qualities (reflection coefficients) of the signal generator and the receiver.

Figure A.2 shows the relationships between attenuation, insertion loss, substitution Loss, and S-parameters (S_{21}).

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