Vol. 23 No. 112 Journal of the Radio Research Laboratories November 1976
Printed in Tokyo, Japan pp. 255–265

OBSERVED TIME DISCONTINUITY OF CLOCK SYNCHRONIZATION IN ROTATING FRAME OF THE EARTH

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(Received on June 22, 1976)

ABSTRACT

In August 1975, a high precision time comparison via a geostationary satellite was made between the atomic clocks in Japan and in the U.S.A. by a two-way mode in SHF band. In this experiment, the Spread Spectrum Random Access (SSRA) communication system was used for transferring the time signal in order to get high resolution of measurements of about one nanosecond. In the course of the experiment via satellite, the time difference was also measured through the ground link including a flying-clock method.

On the analysis of data, the light synchronization discontinuity due to the rotation of the earth was confirmed by two experimental results.

First, the periodical small variation was clearly observed in the data via satellite, which variation corresponds well to the calculated one caused by the effect of diurnal drift of the satellite location coupled with the effect of the earth's rotation.

Second, much better consistency of two measured values, the one via satellite and the other through the ground link, is obtained in case the correction calculated for the effect is applied to the data via satellite.

1. Introduction

The precise time utilizable in many scientific fields can now be kept by atomic clock with high stability. In this state, it is needed to develop higher precision techniques of time comparison or synchronization, especially for intercontinental use. Among many satellite techniques, it has been considered that the two-way method in microwave band via a geostationary satellite offers the highest accuracy.

In August 1975, an international clock comparison via the geostationary satellite, ATS-1, was made by the Radio Research Laboratories in close cooperation with the GSFC of NASA and the U.S. Naval Observatory. In this experiment, it was expected to get a precision better than a few nanoseconds and high accuracy of about 10 nanoseconds. The measurements extending over several hours per day were made for 4 days. Although the data showed that the system used in this experiment offered high resolution of about one nanosecond, there were two points unexplained in preliminary data reduction. One is that there exists slow and small variation during an observation period of several hours, the other is that the consistency of two values measured via satellite and through the ground link including a flying-clock measurement was not so satisfactory.

2. Characteristics of the system of the experiment

The experiment was made between the Kashima station of the RRL in Japan and the Rosman station of the NASA in the U.S.A. through the geostationary satellite ATS-1. The overall block diagram of the system is shown in Fig. 1. As the detail has been described in the reference (1), only the outline of the system is given here.

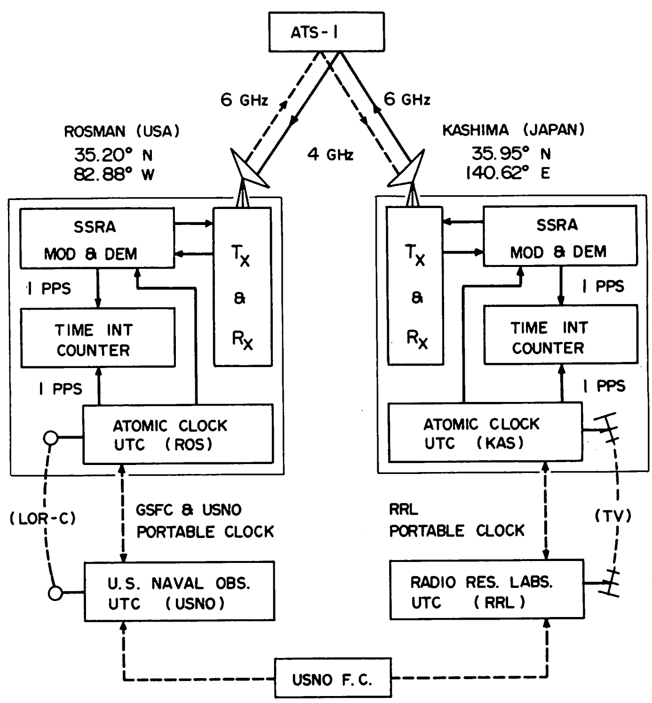

Fig. 1. Block diagram of time comparison experiment

The principle of time comparison by a two-way method is expressed as follows;

$$\frac{1}{2}(M_K - M_R) = \Delta t + \frac{1}{2}(P_{RK} - P_{KR}) \quad \text{.......} \quad (1)$$

where Δt: Time difference between two clocks,

M_K, M_R: Time difference between the clock pulse at a station and the received pulse, measured at the Kashima station and at the Rosman station, respectively,

P_{KR}, R_{RK}: Total time delay through the link from Kashima to Rosman and vice versa respectively, which includes the delay in the system at the ground stations and on satellite, and the propagation delay in space.

As seen from Eq. (1), the accuracy of measurement depends on how accurately the difference of total time delays for both directions can be determined. With a view to getting higher accuracy and precision, the following are worth special mention for this experiment;

(a) Use of the SSRA system: The Spread Spectrum Random Access (SSRA) system is one of the code division multiple access communication systems, in which a pseudo noise (PN) code of 16.376 Mbits per second is used to spread the spectrum of carrier wave to the bandwidth of about 30 MHz as well as to identify the station. For time transfer experiment, this spreading PN code yields fine timing frame, and the transmission of 1 pps signal of the reference clock can be performed by making the phase of reference signal of the SSRA system coherent with that of signal from the clock. Thus, the use of the SSRA system offers two merits in addition to the basic feature of a two-way method. One of them is that the high resolution of measurement better than a few nanoseconds is expected, because the time signal detection can be made with better signal to noise ratio by the delayed-loop correlation detector. The other is that the error in a two-way method can be minimized as the transmission and the reception at both stations can be made simultaneously through the same channel in a repeater on satellite.

(b) High stability of the reference clock: In order to make measurements as precise as possible, the reference time scale at each station was kept by a cesium atomic clock equipped with a high performance beam tube, which has the highest stability among the commercial products available at present. That is, the frequency stability for the averaging time of around several hours is about 6×10^{-14} or better. Furthermore, the reference clock was monitored through the ground link, such as TV or Loran-C signal and a portable clock measurement, with respect to more stable time scale kept by the U.S. Naval Observatory or by the Radio Research Laboratories.

(c) Precision measurement of the delay time: The delays in the transmitter and the receiver were precisely measured at both stations. Then, the error caused by the difference between the system delays for two directions can be corrected with an accuracy of around 10 nanoseconds (1σ).

3. Preliminary considerations on data reduction

Each value of the data is usually an average of 240 measurements, namely for about 4 minutes, and the standard deviation of these measurements was about one nanosecond. Thus, the high resolution of the system was verified experimentally.

All of the measured values obtained during 4 days are plotted in Fig. 2, where the straight line is the regression line for all data, and the standard deviation around this line was 1.7 nanoseconds. It seems, however, that there exists a small and slow variation in the measured data, in particular for those obtained on August 26–27. One of the possible causes is the refraction effect in the ionosphere, but it was difficult to explain the data observed on the other days.

As to the accuracy, the result obtained through the ground link showed that the Kashima clock was 9.42 ± 0.2 microseconds faster than the Rosman clock at $02^h\ 34^m$ UTC on August 27, 1975, while the corresponding value obtained via the satellite link was 9.106 microseconds. The difference between the value measured via satellite and that measured through the ground link was 0.314 microsecond. This is larger than that we expected from a

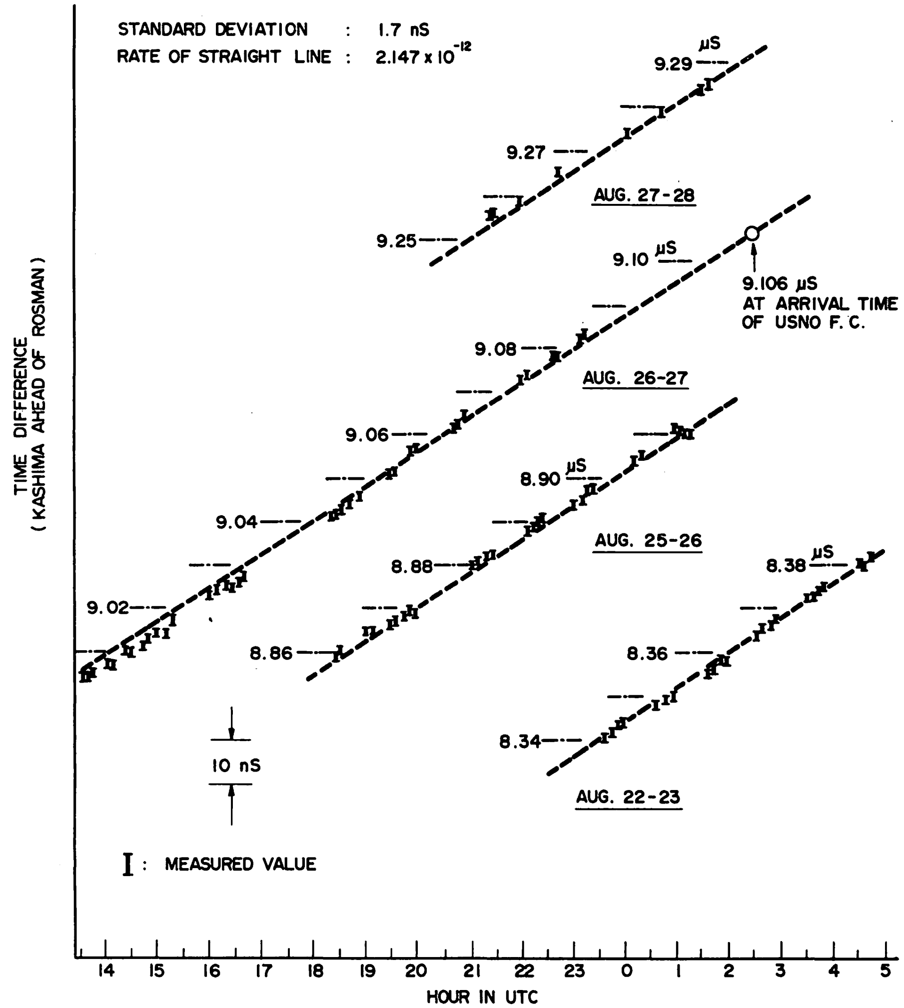

Fig. 2. Measured time difference via satellite

view point of high accuracy to be obtained by the system, even if the uncertainty of 0.2 microsecond caused by a flying-clock measurement is taken into consideration.

Then, the effect of the earth's rotation was examined as one of the reasons for causing these two problems, that is, the existence of small and periodical variation and an insufficient result in accuracy.

With relation to the rotation of the earth, it may be considered that a diurnal change of the distance between a clock rest on the earth and the sun or the moon will cause a variation of the frequency of an atomic clock due to the general relativistic effect. The frequency shift due to this noon-midnight effect is so small(2) that a variation in the time difference between two clocks can be neglected in our experiment. Furthermore, this effect does not cause an offset error except negligible small diurnal variation in the accuracy of clock comparison.

4. Light propagation in a rotating coordinate

It has been known that the experimental evidences for a directional dependency of light propagation in a rotational coordinate were made firstly by Sagnac in 1913, and by Michelson and Gale in 1925. The former was an experiment on a rotating plate and the latter demonstrated the effect of the rotation of the earth, both of which were the optical experiment to detect the fringe shift caused by the rotation. As a result, it became clear that the observed fringe shift is proportional to the angular rate of the rotation and also to the area enclosed by the light path. These experiments were reviewed in detail by Post in the first part of the reference (3), entitled "Sagnac effect".

From this point of view, it is easily understood that the system arrangement of time comparison via a geostationary satellite is very similar to those old experiments, and the only difference is the instrument and technique used in measurement.

In consideration of the propagation delay of radio signals via a geostationary satellite, we can assume that the effect of the gravitational potential of the earth is small and also to be cancelled out by a two-way method, and that the other relativistic effects, such as the Lense-Thirring effect due to the earth's rotation and other cross effects, are to be negligibly small. On this assumption, the problem can be treated in Minkowski space-time coördinate.

The metric in Minkowski space is expressed by

$$ds^2 = -c^2 d\tau^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 \quad \cdots\cdots (2)$$

Eq. (2) can be transformed into that for a rotating coordinate (x', y', z) with an angular rate of ω as follows[(4)]:

$$ds^2 = -[1 - \omega^2 (x'^2 + y'^2)/c^2]\, c^2 dt^2 + 2\omega (x'dy' - y'dx')\, dt + (dx')^2 + (dy')^2 + (dz)^2 \quad \cdots\cdots (3)$$

For light propagation in vacuum, the condition, $d\tau = 0$ or $ds = 0$, can be put into Eq. (3). Then, we can find the solution from the quadratic equation of dt on the assumption, $\omega^2 (x'^2 + y'^2) \ll c^2$, as follows:

$$dt_{1,2} = \pm [1 - \omega^2 (x'^2 + y'^2)/c^2]^{-\frac{1}{2}} [(dx')^2 + (dy')^2 + (dz)^2]^{-\frac{1}{2}} / c + \omega (x'dy' - y'dx')/c^2 \quad \cdots\cdots (4)$$

In order to know the propagation time between two points, we should integrate dt_1 or dt_2 for a finite path according to the direction, such as the route of K-S-R or the route R-S-K as shown in Fig. 3. As the second term of the right-hand side in Eq. (4) causes the difference between the propagation times for two directions, the error occurring in a two-way method, E_p, is expressed as follows:

$$E_p = \frac{1}{2}(t_1 - t_2) = \frac{1}{2}\left[\int_{\substack{\text{path}\\ \text{K to R}}} dt_1 - \int_{\substack{\text{path}\\ \text{R to K}}} dt_2\right]$$

$$= \frac{\omega}{c^2} \int_{\substack{\text{path}\\ \text{K to R}}} (x'dy' - y'dx') = 2\omega A/c^2 \quad \cdots\cdots (5)$$

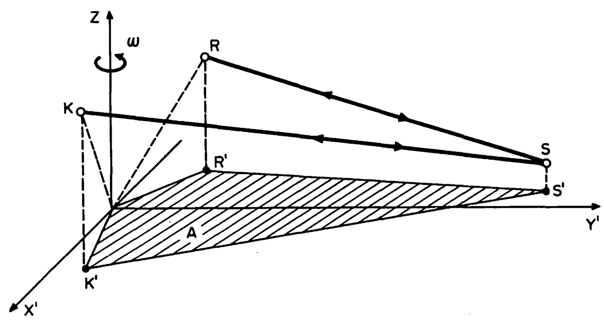

Fig. 3. Signal path via satellite

where A is a loop area projected on $x'-y'$ plane or the equatorial plane in our experiment as shown in Fig. 3. As to this error, the same result obtained in different manners has recently been shown in the references (5) and (6).

It is of interest to note that the loop area A was 866 cm^2 on a rotating plate of about 2 r.p.m. in Sagnac's experiment and about 0.2 km^2 on the earth's rotation in the experiment by Michelson and Gale, and it is about 2×10^8 km^2 for an intercontinental experiment via a geostationary satellite.

5. Reduction of the data measured via satellite

According to Eq. (5), the error due to the effect of the earth's rotation can be calculated in our experiment by the use of the orbital data for the geostationary satellite ATS-1, as shown in Fig. 4. As seen in the Fig. 4 (d), the error varies with the amplitude of about 2 or 3 nanoseconds around the mean value of about 328 nanoseconds, because the loop area, A, varies slowly according to the small diurnal drift of the location of the geostationary satellite. Since the amount and phase of the drift do not change so much for a period of a week, we used the value obtained on August 26 for the whole period of the experiment.

In comparison of the measured data with the calculated values, the following processes are made.

(a) Though we used an atomic clock having the highest stability, it is difficult to consider that the time difference between two atomic clocks changed linearly for the whole period, because we should treat the data with high accuracy of the order of one nanosecond. Then, the reduction of the data is made day by day, as the frequency difference of two clocks for less than 10 hours is expected to be almost constant.

(b) In practice, the correction of the calculated error caused by the effect of the earth's rotation is applied to the measured data, and then a regression line of time difference for the data for each day is determined, which line is considered to be the most probable value of time difference between two remote clocks.

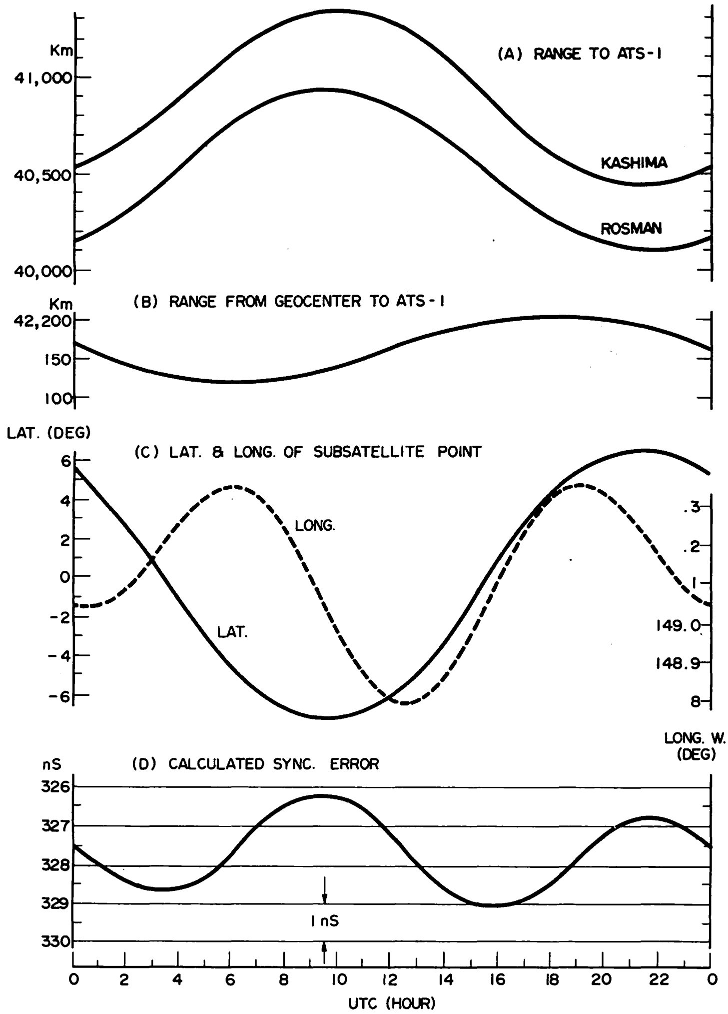

Fig. 4. Orbital data & calculated synchronization error.

The numerical results thus obtained for 4 days are summarized in Table 1 and shown in Fig. 5. The measured small variation around the most probable linear change of time difference is well coincident in amplitude and in phase with the calculated one as shown by the dotted curve in Fig. 5. The effects of the ionosphere appeared about sunrise at Kashima in Japan and about sunset at Rosman in the U.S.A., especially in the data obtained on August 25 to 26.

The frequency difference or the rate of time difference between two clocks is listed in the fourth column of Table 1. The day-to-day change in the relative frequency difference was in a range of -0.8 to $+1.7 \times 10^{-13}$, which is conceivably possible in view of stability of

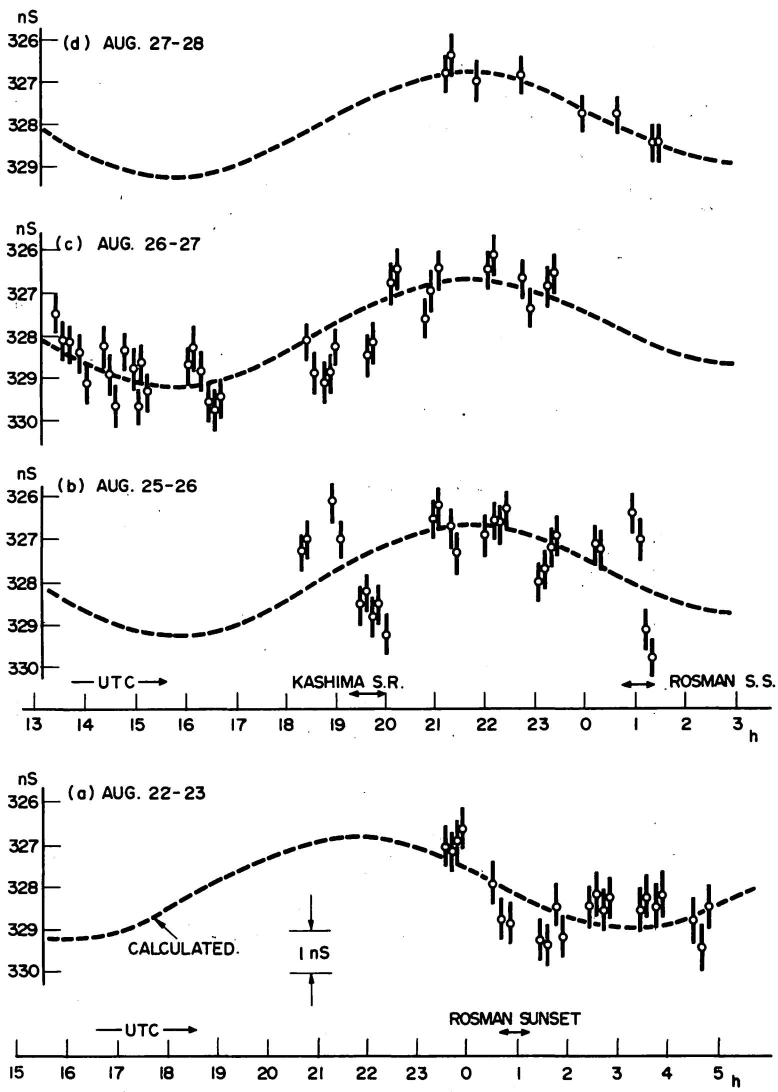

Fig. 5. Observed variation of synch. error.

atomic clocks used in the experiment. However, it surely includes a daily change of the effect of the ionosphere to some extent, but it was difficult to determine it quantitatively as mentioned below.

The degree of coincidence between the measured value and the calculated one concerning a small variation due to the drift of satellite location is shown by a correlation coefficient in the last column of Table 1. As there is much correlation between them, it is assured that the correction for the effect of the earth's rotation should be applied to the measured value via satellite.

Table 1. Results of Data Analysis

Date Aug. 1975	Number of data	Observation time in hrs	Observed frequency difference ($\times 10^{-13}$)	Standard error (in ns)	Correlation coefficient
22-23	22	5	22.84	0.64	0.65
25-26	27	7	21.43	1.01	0.31
26-27	37	10	22.26	0.57	0.85
27-28	8	4.5	20.60	0.27	0.97
				For total data of 94 :	0.74
				For data of 67 (except Aug. 25-26) :	0.83

As shown in the fifth column of Table 1, the standard error of these measurements around the calculated values was about 0.6 nanosecond for the whole period except on August 25–26, when the large effect of the ionosphere was clearly observed.

As the additional ionospheric delay is frequency dependent, an error in a two-way method will be caused by the unequality of the propagation time for two directions of the path. In high frequency approximation, the ionospheric delay, d, is

$$d = \frac{40.3}{cf^2} \int_{path} N_e \, ds = \frac{40.3}{cf^2} N \quad \ldots\ldots (6)$$

where
- c: light velocity,
- f: frequency in Hz,
- Ne: electron density per meter cube,
- N: total electron content along the path.

Using Eqs. (1) and (6), the error due to the ionospheric effect, E, for the up-link of 6 GHz and the down-link of 4 GHz is given by

$$E = \frac{1}{2}(d_{RK} - d_{KR}) \simeq 2.33 \times 10^{-27}(N_R - N_K) \quad \ldots\ldots (7)$$

where N_R, N_K; total electron contents (el/m^2) along the path from satellite to the Rosman station and from satellite to the Kashima station, respectively.

As seen in Eq. (7), the error is proportional to the difference of the total electron contents in Rosman site and in Kashima site. In the data obtained on August 25–26, there exist the changes of about 3 nanoseconds at sunrise of Kashima and at sunset of Rosman, which correspond to the difference of the total electron contents of about 1.3×10^{18} el/m^2. The total electron contents will be subjected to considerable variations, such as diurnal, day-to-day, seasonal and sunspot cycle, and also depends on the elevation angle of the path. As the difference of local time for two stations is about 10 hours, there must be a diurnal change in the error, but it was not noticeable from the results. It might as well be said that the experiment was conducted in summer, when the difference of the electron content between day and night is minimum in a year, and also in the year of 1975 the sunspot cycle was at the minimum in addition to rather quiet condition of the ionosphere for the period of experiment.

With the data reduction mentioned above, the time difference measured by satellite becomes 9.439 microseconds at 02^h34^m on August 27, when the flying-clock measurement of the U.S. Naval Observatory was made. Thus, the difference of the values measured via satellite and by the ground link becomes -0.02 ± 0.2 microsecond, instead of 0.31 ± 0.2 microsecond in the case without correction for the effect of the earth's rotation.

It is necessary, however, to estimate an amount of the relativistic effect on a flying clock, because the correction was not applied in the reduction of the USNO flying-clock measurement(7). The effect includes three terms, which are the gravitational shift depending upon an altitude, the second-order doppler shift caused by ground speed and the kinematic shift due to the rotation of the earth mentioned above.

Assuming that the flight was at an altitude of 10 km and a ground speed of 900 km per hour along the great circle path between Washington D.C. to Tokyo via Hawaii, the flying clock would gain by 87 nanoseconds for the trip from east to west and would lose by 4 nanoseconds for the return trip to eastward. As the measurement of the USNO flying clock was made at the RRL on the second day at the beginning of 24-day round trip, it may be estimated that the correction would be 80 nanoseconds.

Applying this estimated correction to the measured value obtained through the ground link, the time difference between the station clocks becomes 9.50 ± 0.2 microseconds, which value should be compared with the value of 9.439 microseconds obtained via satellite link corrected for the rotational error. The difference between these two values obtained through different links is 0.06 ± 0.2 microsecond, which is considered to be reasonable in view of an uncertainty of 0.2 microsecond in the flying-clock measurement performed.

6. Conclusion

In the clock comparison experiment via a geostationary satellite, the light synchronization discontinuity due to the rotation of the earth was experimentally confirmed by two results as follows:

(a) The periodical small variation was clearly observed in the data via satellite, which corresponds exactly to the one calculated from the effect of the diurnal drift of the satellite location coupled with the effect of earth's rotation.

(b) Much better consistency of two values measured via satellite and through the ground link is obtained in case the correction calculated for the effect is applied to the measured value via satellite.

It is noted that the observation of the effect on the light synchronization by this experiment corresponds to that of the same effect on an atomic clock flying around the world to eastward and to westward examined by Hafele and Keating(8).

The error on a two-way measurement caused by the refraction of the ionosphere should be studied in detail in order to get higher precision and accuracy in future experiments.

7. Acknowledgement

The author wishes to thank Professor H. Hirakawa of Tokyo University for his advice and encouragement in analyzing the data in the satellite experiment, and Dr. G.M.R. Winkler of the U.S. Naval Observatory, A.R. Chi and C. Wardrip of the GSFC of NASA for their many helps on portable clock measurements.

References

(1) M. Yamamoto, K. Harada and Y. Saburi, "A Time Comparison Experiment Performed by SSRA System Via ATS-1", Jour. Radio Research Labs, Vol. 23, No. 110, March 1976.
(2) B. Hoffmann, "Noon-Midnight Red Shift", Phys. Rev. Vol. 121, No. 1, Jan. 1, 1961.
(3) E.J. Post, "Sagnac Effect", Rev. Mod. Phys. Vol. 39, No. 2, April 1967.
(4) H. Hirakawa, "Relativity", (in Japanese) Kyoritsu Publishing Co., 1971.
(5) V. Reinhardt, "Relativistic Effects of the Rotation of the Earth on Remote Clock Synchronization", Proc. 6th Annual PTTI Meeting, Dec. 1974.
(6) N. Ashby, "An Earth-Based Coordinate Clock Network", NBS Tech. Note 659, April 1975.
(7) G.M.R. Winkler, Private Communication.
(8) J.C. Hafele and R.E. Keating, "Around the World Atomic Clock: Observed Relativistic Time Gains", Science, Vol. 117, July 14, 1972.