7-2 Continuous variable teleportation of non-classical states in noisy environment

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Transmission of nonclassical quantum states by quantum teleportation of continuous variables is studied. Protocol of quantum teleportation via a two-mode squeezed-vacuum state in a noisy environment is formulated by the Glauber-Sudarshan *P*-function. Using the nonclassical depth as an estimation parameter of transmission performance, we compare the teleportation scheme with the direct transmission through a noisy channel. The noise model is based on the coupling to the vacuum field. We find that the teleportation channel has better transmission performance than the direct transmission channel in a certain region. The bounds for such region and for obtaining the nonvanished nonclassicality of the teleported quantum states are also discussed. We also mention the required conditions for transmitting nonclassical features in real experiments.

Keywords
Quantum teleportation, Continuous variable, Two-mode squeezed-vacuum, Non-classical depth

1 Introduction

Quantum information technologies, including quantum computation and quantum cryptography, can be realized by completely controlling the quantum states. It is well known that these technologies have some amazing performances compared to the present technologies based on classical physics in which quantum theory is not included[1]. To construct the systems or devices for these technologies, it is necessary to establish the theory for the transformation properties of quantum state itself, *i.e.* "quantum information", as conventional information theory has been used for conventional information processing technologies.

Meanwhile, much attention has recently been paid to the scheme of "quantum teleportation" that is the protocol to transform quantum states indirectly. The name of "teleportation" has come from the following reason. Here, we consider the reconstruction of the prepared arbitrary quantum state at a distant place without directly transmitting the prepared state itself. In the region of classical theory, it is a trivial task. Measuring the parameters that are necessary for reconstructing the state, transferring the results of measurements, and then reconstructing the original state at distant places. On the other hand, it seems impossible to do the same task in the quantum region because of the following two reasons. First, it is impossible to know exact values of quantum parameters by the only one-time measurement since these are given by a probabilistic function. Secondly, to make plural perfect copies of the original state is also prohibited by the principle of quantum mechanics [2]. Nevertheless, in 1993, Bennett *et al.*[3] showed that such indirect transformation of quantum state is possible by using the quantum mechanically entangled state, which has no correspondence in classical theory, and so-called simultaneous measurement. They called this protocol as "quantum teleportation". In their original proposal, the theory was treated within finite dimensions in Hilbert space. Then it was generalized into the teleportation of continuous variables by using continuously entangled states[4]. More practical scheme of the continuous variable teleportation was proposed[5] in which a two-mode squeezed-vacuum state is employed as an entangled state. The experimental demonstration of a coherent state teleportation was performed by using quantum optical fields[6].

While quantum teleportation attracts a great deal of researcher's interests as mentioned above, its performance in the viewpoint of quantum information transmission has not been clarified yet. Although the teleportation of a coherent state has several advantages in experimental point of view, it is fundamentally able to transfer arbitrary unknown quantum states including a variety of nonclassical states by teleportation. In this report we investigate that how much nonclassicality can be transferred by the noisy teleportation of continuous variables and if the capability of the teleportation is better than that of the direct transmission or not[7], with the help of the Glauber-Sudarshan *P*-function representation and the nonclassical depth which has been proposed to estimate the strength of the nonclassicality[8]. In our model, it is assumed that the noise comes from the coupling between the system and an environment in the vacuum state, which is commonly encountered in optical quantum communication networks. We reveal that the transfer capability of nonclassicality by the teleportation strictly depends on the degree of the two-mode squeezing, the loss of the channel, and the strength of the initial noncassicality of the quantum state to be teleported. It is shown that the teleportation channel has better transmission performance than the direct transmission in a certain region.

In the following sections, we precisely discuss these topics by using equations of quantum mechanics.

2 Protocol of the Teleportation of Continuous Variables

A schematic of the continuous variable teleportation is depicted in Fig.1[5][6]. Teleportation between the sender, Alice, and the receiver, Bob, is performed by sharing a two-mode squeezed-vacuum state $|\Psi_{SV}^{AB}\rangle$ given by

$$\begin{aligned}|\Psi_{SV}^{AB}\rangle &= \exp\left[r(\hat{a}^\dagger\hat{b}^\dagger - \hat{a}\hat{b})\right]|0^A\rangle\otimes|0^B\rangle \\ &= \sqrt{1-\lambda^2}\sum_{n=0}^{\infty}\lambda^n|n^A\rangle\otimes|n^B\rangle,\end{aligned} \tag{1}$$

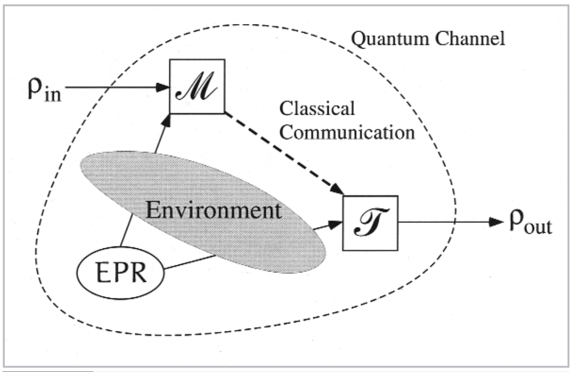

Fig.1 *Continuous variable quantum teleportation*

In the figure,'M'stands for the quantum measurement performed at the sender side,'T'represents the unitary transformation carried out at the receiver side and'EPR'indicates the entangled quantum state shared by Alice and Bob.

where $\hat{a}(\hat{b})$ and $\hat{a}^\dagger(\hat{b}^\dagger)$ are the bosonic annihilation and creation operators for the mode A (B), respectively. $|n^A\rangle$ and $|n^B\rangle$ are the photon-number eigenstates of the mode A and B, respectively, and the parameter λ is defined by $\lambda = \tanh r$. For the sake of simplicity, the squeezing parameter r in Eq. (1) has been assumed to be positive through this paper. The mode A and B are assigned to the modes for Alice and Bob, respectively.

In a realistic situation, since the environment inevitably in uences the two-mode squeezed-vacuum shared by Alice and Bob, the pure squeezed-vacuum state is turned into the mixed state and the quantum entanglement is degraded. A state change of the quantum states induced by the environment is fully described by a completely positive (CP) map [9]. Thus the mixed quantum state ρ_{SV}^{AB} shared by Alice and Bob is represented by the following expression:

$$\hat{\rho}_{SV}^{AB} = \left(\hat{\mathcal{L}}^A\otimes\hat{\mathcal{L}}^B\right)|\Psi_{SV}^{AB}\rangle\langle\Psi_{SV}^{AB}|, \tag{2}$$

where $\hat{\mathcal{L}}^A$ and $\hat{\mathcal{L}}^B$ are the CP maps for the mode A and B, respectively, and we consider the situation that these CP maps have the same properties. The environment is assumed to be in the vacuum state since thermal photons can be neglected in optical frequency region. Under these assumptions, the CP maps $\hat{\mathcal{L}}^A$ and $\hat{\mathcal{L}}^B$ are given by[10]

$$\hat{\mathcal{L}}^{A,B} = \exp\left[g\left(\hat{\mathcal{K}}_-^{A,B} - \hat{\mathcal{K}}_0^{A,B} + \frac{1}{2}\right)\right] \tag{3}$$

where g is a positive parameter and the superoperators $\hat{\mathcal{K}}_-^A$ and $\hat{\mathcal{K}}_0^A$ are defined by

$$\hat{\mathcal{K}}_-^A\hat{X} = \hat{a}\hat{X}\hat{a}^\dagger \quad \hat{\mathcal{K}}_0^A\hat{X} = \frac{1}{2}(\hat{a}^\dagger\hat{a}\hat{X} + \hat{X}\hat{a}^\dagger\hat{a} + \hat{X}), \tag{4}$$

for an arbitrary operator $\hat{X}$, and $\hat{\mathcal{K}}_-^B$ and $\hat{\mathcal{K}}_0^B$ follow the same definitions with $\hat{b}$ and $\hat{b}^\dagger$. The CP maps $\hat{\mathcal{L}}^A$ and $\hat{\mathcal{L}}^B$ transform a coherent state into another coherent state with a reduced complex amplitude such as

$$\hat{\mathcal{L}}|\alpha\rangle\langle\beta| = E(\alpha,\beta)|\alpha\sqrt{T}\rangle\langle\beta\sqrt{T}|, \tag{5}$$

where $T = \exp(-g)$ and $E(\alpha, \beta)$ is the function

$$E(\alpha,\beta) = \exp\left[-\frac{1}{2}(1-T)\left(|\alpha|^2 + |\beta|^2 - 2\alpha\beta^*\right)\right]. \tag{6}$$

The parameter T represents the transmittance of the noisy quantum channel. Although this is one of the simplest loss mechanism in quantum channels, it can model experimental situations well.

Suppose that Alice has an arbitrary quantum state $\hat{\rho}_{\rm in}^C$ which is to be teleported to Bob's hand. The operator $\hat{\rho}^{ABC} = \hat{\rho}_{\rm SV}^{AB} \otimes \hat{\rho}_{\rm in}^C$ represents the total quantum state of Alice and Bob. To teleport the quantum state $\hat{\rho}_{\rm in}^C$, Alice performs the simultaneous measurement of the position and the momentum of the mode A and C[12] described by the projection operator $\hat{X}^{AC}(x, p) = |\Phi^{AC}(x, p)\rangle\langle\Phi^{AC}(x, p)|$. The vector $|\Phi^{AC}(x, p)\rangle$ is the simultaneous eigenstate of $\hat{x}^C - \hat{x}^A$ and $\hat{p}^C + \hat{p}^A$,

$$|\Phi^{AC}(x,p)\rangle = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} {\rm d}y\, |x^C + y^C\rangle \otimes |y^A\rangle {\rm e}^{{\rm i}py}. \tag{7}$$

The probability $P(x, p)$ that Alice obtains the measurement outcome (x, p) is given by

$$P(x,p) = {\rm Tr}_{ABC}\left[\left(\hat{X}^{AC}(x,p)\otimes\hat{1}^B\right)\left(\hat{\rho}_{\rm SV}^{AB}\otimes\hat{\rho}_{\rm in}^C\right)\right]. \tag{8}$$

Alice informs Bob of her measurement outcome (x, p) by a classical communication channel. By using the state-reduction formula [9], the quantum state $\hat{\rho}^B(x, p)$ at Bob's hand becomes

$$\hat{\rho}^B(x,p) = \frac{{\rm Tr}_{AC}\left[\left(\hat{X}^{AC}(x,p)\otimes\hat{1}^B\right)\left(\hat{\rho}_{\rm SV}^{AB}\otimes\hat{\rho}_{\rm in}^C\right)\right]}{{\rm Tr}_{ABC}\left[\left(\hat{X}^{AC}(x,p)\otimes\hat{1}^B\right)\left(\hat{\rho}_{\rm SV}^{AB}\otimes\hat{\rho}_{\rm in}^C\right)\right]}. \tag{9}$$

After receiving the Alice's measurement outcome (x, p), Bob applies the unitary operator $\hat{D}^B(x, p) = {\rm e}^{i(p\hat{x}^B - x\hat{p}^B)} = {\rm e}^{\mu\hat{b}^\dagger - \mu^*\hat{b}}$ to the quantum state $\hat{\rho}^B(x, p)$ where $\mu = (x+ip)/\sqrt{2}$. Then he finally obtains

$$\hat{\rho}_{\rm out}^B(x,p) = \hat{D}^B(x,p)\hat{\rho}^B(x,p)\hat{D}^{B\dagger}(x,p). \tag{10}$$

Averaging the output $\hat{\rho}_{\rm out}^B(x, p)$ over the probability distribution of $P(x, p)$ in Eq. (8), the averaged output state for Bob $\hat{\rho}_{\rm out}^B$ is derived as

$$\begin{aligned}\hat{\rho}_{\rm out}^B &= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dp\, P(x,p)\hat{\rho}_{\rm out}^B(x,p)\\ &= \int_{-\infty}^{\infty} {\rm d}x \int_{-\infty}^{\infty} {\rm d}p\, \hat{D}^B(x,p)\\ &{\rm Tr}_{AC}\left[\left(\hat{X}^{AC}(x,p)\otimes\hat{1}^B\right)\left(\hat{\rho}_{\rm SV}^{AB}\otimes\hat{\rho}_{\rm in}^C\right)\right]\hat{D}^{B\dagger}(x,p).\end{aligned} \tag{11}$$

3 Formulation based on the *P*-function Representation

An arbitrary quantum state can be represented in a diagonal form with respect to coherent states, which is the *P*-function representation[10]. In the following section, we formulate the teleportation protocol based on the *P*-function representation which provides us some physical insights and the most straightforward formulation in order to quantify the transmission of the nonclassicality of the input quantum states with respect to the nonclassical depth. It is well known that when the *P*-function is singular or not positive definite, the quantum state is nonclassical.

The *P*-function representation of the arbitrary input quantum state $\hat{\rho}_{\rm in}^C$ is given by

$$\hat{\rho}_{\rm in}^{C} = \int \mathrm{d}^2\alpha\, P_{\rm in}(\alpha)|\alpha^C\rangle\langle\alpha^C|. \tag{12}$$

It is clear from Eq. (12) that if the teleported output state for a coherent state input is found, the teleported quantum state for an arbitrary input is automatically given. After tedious calculations, we obtain the teleported quantum state $\hat{\rho}_{\rm out}^{B}(x,p)$ as

$$\hat{\rho}_{\rm out}^{B}(x,p) = \frac{\int \mathrm{d}^2\alpha\, P_{\rm in}(\alpha)\mathrm{e}^{-\frac{1-\lambda^2}{1-\lambda^2(1-T)}|\alpha-\mu|^2}\hat{D}^B(\mu_{\alpha\lambda T})\hat{\rho}^B_{\bar{n}_{\lambda T}}\hat{D}^{B\dagger}(\mu_{\alpha\lambda T})}{\int \mathrm{d}^2\alpha\, P_{\rm in}(\alpha)\exp\left[-\frac{1-\lambda^2}{1-\lambda^2(1-T)}|\alpha-\mu|^2\right]}. \tag{13}$$

Thus the teleported quantum state averaged over the probability distribution $P(x, p)$ is given by

$$\begin{aligned}\hat{\rho}_{\rm out}^{B} &= \int_{-\infty}^{\infty}\mathrm{d}x\int_{-\infty}^{\infty}\mathrm{d}p\, P(x,p)\hat{\rho}_{\rm out}^{B}(x,p)\\ &= \int \mathrm{d}^2\alpha\, P_{\rm in}(\alpha)\hat{D}^B(\alpha)\hat{\rho}^B_{\bar{n}_{\lambda T}}\hat{D}^{B\dagger}(\alpha),\end{aligned} \tag{14}$$

where the density operator $\hat{\rho}^B_{\bar{n}_{\lambda T}}$ represents the thermal state with average photon number $\bar{n}_{\lambda T}$ which is given by

$$\bar{n}_{\lambda T} = 1 - \frac{2\lambda T}{1+\lambda} = 1 - \left(1-\mathrm{e}^{-2r}\right)T. \tag{15}$$

For example, the output state for the coherent state input $P_{\rm in}(\beta) = \delta^{(2)}(\alpha-\beta)$ is given by $\hat{\rho}^B_{\rm out} = \hat{D}^B(\alpha)\hat{\rho}^B_{\bar{n}_{\lambda T}}\hat{D}^{B\dagger}(\alpha)$ which represents the thermal coherent state. More generally, the basis of the coherent state expansion $(|\alpha\rangle\langle\alpha|)$ in the input state $\hat{\rho}^C_{\rm in}$ is transformed into the thermal coherent state $\hat{D}(\alpha)\hat{\rho}_{\bar{n}_{\lambda T}}\hat{D}^{\dagger}(\alpha)$ by the lossy channel teleportation. Here we note that the environment in the teleportation channel does not directly degrade the transferred quantum state $\hat{\rho}_{\rm in}$. Degradation of $\hat{\rho}_{\rm in}$ in this teleportation model is caused by the imperfect squeezing and the environmental decoherence of the two-mode squeezed-vacuum before the displacement operation by Bob. As a consequence, the losses of the teleportation channel injects the thermal noise into the transferred quantum state with the average photon number $\bar{n}_{\lambda T}$ instead of the direct degradation of $P_{\rm in}(\alpha)$ in Eq. (14). It characterizes the teleportation channel definitely different from the direct transmission channel as discussed later.

The teleportation channel is also characterized by the following transformation of the P-function;

$$P_{\rm in}(\alpha) \longrightarrow P_{\rm out}(\alpha) = \frac{1}{\pi\bar{n}_{\lambda T}}\int \mathrm{d}^2\beta\, P_{\rm in}(\beta)\mathrm{e}^{-\frac{|\alpha-\beta|^2}{\bar{n}_{\lambda T}}}, \tag{16}$$

where $P_{\rm out}(\alpha)$ is the P-function of the quantum state $\hat{\rho}_{\rm out}$ and this transformation simply shows the thermalization process of the teleportation.

4 Fidelity

For the teleportation of the pure input state $\hat{\rho}^C_{\rm in} = |\psi^C\rangle\langle\psi^C|$, the fidelity is given by

$$\begin{aligned}F^{\rm tel} &= \langle\psi^C|\hat{\rho}^B_{\rm out}|\psi^C\rangle\\ &= \frac{1}{\bar{n}_{\lambda T}}\int \mathrm{d}^2\alpha\int \mathrm{d}^2\beta\, Q_{\rm in}(\alpha)P_{\rm in}(\beta)\mathrm{e}^{-\frac{|\alpha-\beta|^2}{\bar{n}_{\lambda T}}},\end{aligned} \tag{17}$$

where $Q_{\rm in}(\alpha) = (1/\pi)|\langle\alpha^C|\psi^C\rangle|^2$ is the Q-function of the original state. It is easy to see that

$$\lim_{T\to 1}\lim_{\lambda\to 1}F^{\rm tel} = 1 \quad \lim_{\lambda\to 0}F^{\rm tel} = \pi\int \mathrm{d}^2\alpha\, Q^2_{\rm in}(\alpha) < 1. \tag{18}$$

When the original state is the coherent state, the fidelity becomes

$$F^{\rm tel}_{\rm coh}(\bar{n}_{\lambda T}) = \frac{1}{1+\bar{n}_{\lambda T}} = \frac{1+\lambda}{2(1+\lambda-\lambda T)}. \tag{19}$$

In case of the ideal quantum teleportation (T = 1), this result is identical with that obtained so far[11]. Similarly, the fidelities for the Fock state $|n\rangle$ and the Schrödinger-cat state $(|\alpha\rangle - |-\alpha\rangle)/\sqrt{2(1-\mathrm{e}^{-2|\alpha|^2})}$ are given by

$$F^{\rm tel}_{n}(\bar{n}_{\lambda T}) = \frac{1}{1+\bar{n}_{\lambda T}}\left(\frac{1-\bar{n}_{\lambda T}}{1+\bar{n}_{\lambda T}}\right)^n P_n\left(\frac{1+\bar{n}^2_{\lambda T}}{1-\bar{n}^2_{\lambda T}}\right), \tag{20}$$

and

$$F^{\rm tel}_{\rm cat}(\bar{n}_{\lambda T}) = \frac{1}{2(1+\bar{n}_{\lambda T})}\left\{1+\left[\frac{\sinh\left(\frac{1-\bar{n}_{\lambda T}}{1+\bar{n}_{\lambda T}}|\alpha|^2\right)}{\sinh(|\alpha|^2)}\right]^2\right\}, \tag{21}$$

respectively, where P_n is the Legendre polynomial of order n. It is worth noting that the fidelities in Eqs. (19), (20) and (21) take finite values even if T = 0 (the input state is completely lost and turned into the vacuum state). This is because the output state can still be made with the finite optical energy at Bob's hand after the classical communication from Alice.

5 Nonclassical Depth

The nonclassicality of quantum states are very important in the fields of quantum optics and quantum information theory. In this section, we briefly follow the definition of the nonclassical depth first, and then investigate the transfer property of the nonclassical depth by teleportation. The nonclassical depth τ_c[8] of the quantum state ρ is defined as the minimum value of the parameter τ which gives the non-negative value of the following quantity $R(\alpha, \tau)$ for all α.

$$R(\alpha,\tau) = \frac{1}{\pi\tau}\int d^2\beta\, P(\beta) \exp\left(-\frac{|\alpha-\beta|^2}{\tau}\right), \qquad (22)$$

where $P(\alpha)$ is the P-function of the quantum state ρ and τ is a real parameter. The nonclassical depth τ_c always satisfy the inequality $0 \leq \tau_c \leq 1$. The Fock state and the superposition of two coherent states (the Schrödinger-cat state) have the nonclassical depth of unity ($\tau_c = 1$). The nonclassical depth of the single-mode squeezed state with squeezing parameter $\xi = re^{i\theta}$ is given by

$$\tau_c = \frac{\tanh|\xi|}{1+\tanh|\xi|} = \frac{1}{2}\left(1 - e^{-2|\xi|}\right), \qquad (23)$$

which takes the maximum value of $\frac{1}{2}$ in the strong squeezing limit.

The nonclassical depth of the teleported quantum state ρ_{out} is easily found from Eq. (14). Substituting Eq. (14) into Eq. (22), the R-function of the teleported quantum state ρ_{out} is calculated as

$$R(\alpha,\tau) = \frac{1}{\pi(\tau+\bar{n}_{\lambda T})}\int d^2\beta\, P_{in}(\beta) \exp\left(-\frac{|\alpha-\beta|^2}{\tau+\bar{n}_{\lambda T}}\right). \qquad (24)$$

This equation shows that the nonclassical depth of the original state τ_c^{in} and that of the teleported state τ_c^{out} are related by the following relation:

$$\tau_c^{out} = \max\left[\tau_c^{in} - \bar{n}_{\lambda T}, 0\right]. \qquad (25)$$

Thus for the teleported quantum state ρ_{out} to keep the nonclassical properties, the squeezing parameter r of the two-mode squeezed-vacuum state shared by Alice and Bob have to satisfy

$$r > -\frac{1}{2}\ln\left(1 - \frac{1-\tau_c^{in}}{T}\right). \qquad (26)$$

If $T \leq 1 - \tau_c^{in}$, none of any nonclassical properties remain in the teleported quantum state. If $r = 0$ (no squeezing), the nonclassical properties will never be teleported, *i.e.*, $\tau_c^{out} = 0$ as expected. Fig.2 shows the nonclassical depth τ_c^{out} of the teleported state ρ^{out} as a function of the squeezing parameter r for several loss parameters T (A), and the lower bound of r for obtaining finite τ_c^{out} at Bob's hand (B). Eq. (25) implies that the transmitting performance

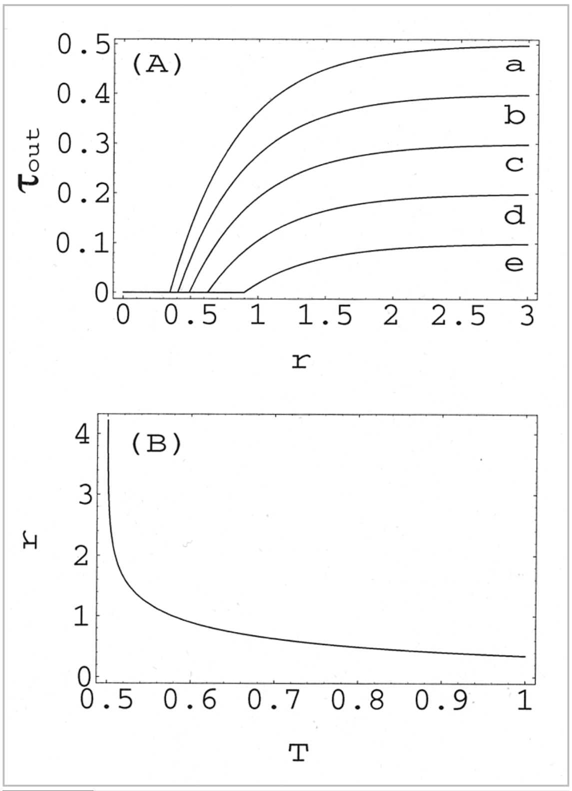

Fig.2 Figure (A) shows the nonclassical depth τ_{out} of the teleported quantum states ρ_{out} in case of (a) $T=1.0$, (b) $T=0.9$, (c) $T=0.8$, (d) $T=0.7$ and (e) $T=0.6$. Figure (B) indicates the lower bound of the squeezing parameter for obtaining the non-vanished nonclassical depth of the teleported quantum state ρ_{out}

In the both figures, the nonclassical depth of the input quantum state ρ_{in} is assumed to be $\tau_{in} = 0.5$.

measured by the nonclassical depth explicitly does not depend on what kind of nonclassical state is to be teleported. It means that if two quantum states have the same nonclassical depth, their teleported quantum states also have the same nonclassical depth, independently of their quantum properties. For example, the Fock state and the Scrödinger-cat state have the same nonclassical depth ($\tau_c = 1$) and thus have the same transfer properties about the nonclassicality while the fidelities in Eq. (20) and Eq. (21) are obviously different.

6 Teleportation and the Direct Transmission

In this section, we discuss the fidelity and the nonclassical depth of the continuous variable teleportation channel and those of the direct transmission channel. It is reasonable to assume that the direct transmission channel is defined by the CP map which is used to transfer the two-mode squeezedvacuum in the teleportation channel (Eq. (3)). By this assumption, the CP map of the direct transmission $\mathcal{L}$ is characterized by the transmittance T. In the above sections, it has been tacitly assumed that the source of the two-mode squeezed-vacuum is located on the middle point of the whole teleportation channel and the two quantum channels for the two-mode squeezed-vacuum have the same length. Thus the CP maps $\mathcal{L}^{A,B}(T)$ and $\mathcal{L}(T^2)$ are used as the channels for the two-mode squeezed-vacuum and the channel for the direct transmission, respectively. We first derive the formula of the fidelity and the nonclassical depth in the direct transmission channel for given T and then compare them with those of the teleportation channel with appropriate lengths.

With the help of Eq. (5), the output state for the direct transmission channel with the transmittance T is calculated as

$$\hat{\rho}_{\text{out}}^{\text{dir}} = \frac{1}{T}\int d^2\alpha\, P_{\text{in}}\left(\frac{\alpha}{\sqrt{T}}\right)|\alpha\rangle\langle\alpha|, \qquad (27)$$

where the input state is given by Eq. (12). The transmission fidelity is also derived in a same manner of the above sections as

$$F^{\text{dir}} = \frac{\pi}{T}\int d^2\alpha\, Q_{\text{in}}(\alpha)P_{\text{in}}\left(\frac{\alpha}{\sqrt{T}}\right). \qquad (28)$$

Equations (27) and (28) clearly show that the P-function of the input state after the direct transmission is directly degraded by the environment while the loss of the teleportaion is the thermalization process as shown in Eqs. (14) and (17). Actually the fidelities for the Fock state and the Schrödinger-cat state transmissions by the direct transmission calculated from Eq. (28) are written by the function of T as

$$F_{\text{cat}}^{\text{dir}}(T) = \left[\frac{\sinh(\sqrt{T}|\alpha|^2)}{\sinh(|\alpha|^2)}\right]^2 \cosh\left((1-T)|\alpha|^2\right), \qquad (29)$$

and

$$F_n^{\text{dir}}(T) = \exp[n\log T] = e^{-gn}, \qquad (30)$$

respectively. These are obviously different from Eqs. (20) and (21). Dependence on T of the fidelities for the teleportation $F_{\text{cat}}^{\text{tel}}(\overline{n}_{\lambda T}(r, T))$ and $F_n^{\text{tel}}(\overline{n}_{\lambda T}(r, T))$ for several r are compared to those of the direct transmission $F_{\text{cat}}^{\text{dir}}(T^2)$ and $F_n^{\text{dir}}(T^2)$ in Fig.3.

The R function for the nonclassical depth of the directly transmitted state is also given by

$$R^{\text{dir}}(\alpha,\tau) = \frac{1}{\pi\tau}\int d^2\beta\, P_{\text{in}}(\beta)\exp\left(-\frac{|\sqrt{T}\beta-\alpha|^2}{\tau}\right), \qquad (31)$$

where T is the transmittance of the direct transmission channel. By considering the definition of the nonclassical depth and $R^{\text{dir}}(\alpha\sqrt{T}, \tau)$, the nonclassical depth for the output state of the direct transmission $\tau_{\text{out}}^{\text{dir}}$ is simply derived as

$$\tau_{\text{out}}^{\text{dir}}(T) = \tau_{\text{in}}T. \qquad (32)$$

The channel always transmit a part of τ_{in} when the input state is nonclassical.

Since the nonclassical depth of the outputs for the teleportation and the direct transmission channels are simply given by Eqs. (25) and (32), respectively, it is now able to compare them analytically. Define the difference between the two kinds of quantum channels

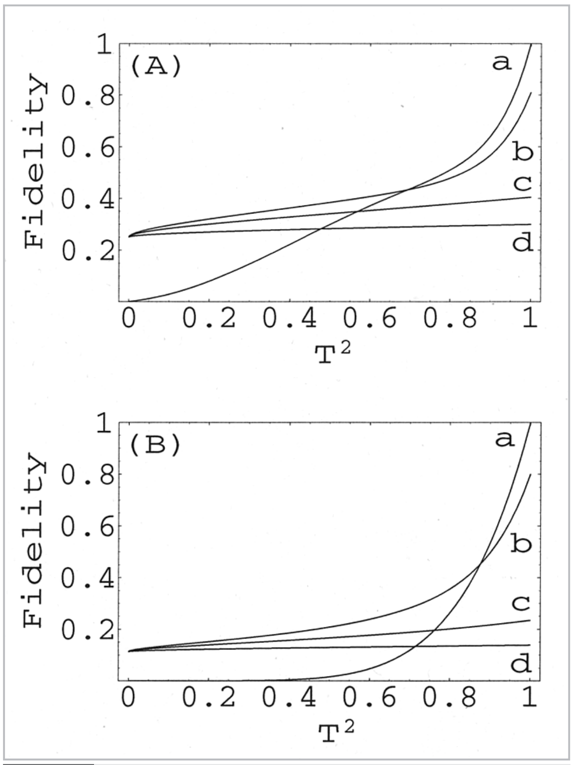

Fig.3 *Dependence on the transmittance T^2 of (A) F_{cat} and (B) F_n for (a) the direct transmission and the teleportation with (b) $r=2.0$, (c) $r=0.7$ and (d) $r=0.2$, respectively, where $\bar{n}=|\alpha|^2=6.0$*

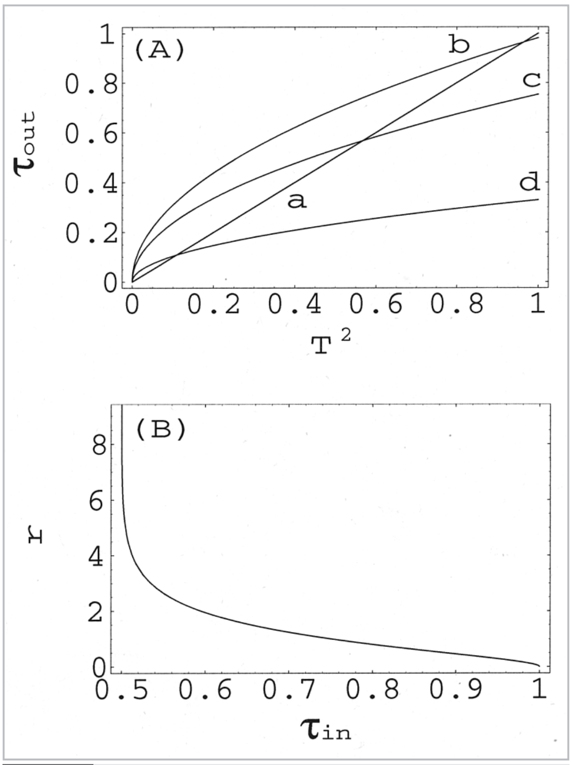

Fig.4 *Figure (A) shows the dependence of the nonclassical depth τ_{out} on the transmittance T for (a) the direct transmission and the teleportation with (b) $r=2.0$, (c) $r=0.7$ and (d) $r=0.2$, respectively, where $\tau_{in}=1.0$. Figure (B) indicates the lower bound for existing the positive $\tau_D(T)$ within $0 \leq T \leq 1$*

$\tau_D(T)$ as

$$\tau_D(T) = \tau_{out}^{tel}(r,T) - \tau_{out}^{dir}(T^2)$$
$$= -\tau_{in}T^2 + (1-e^{-2r})T + \tau_{in} - 1 \quad (0 \leq T \leq 1). \quad (33)$$

The bound for the positive $\tau_D(T)$ is easily found as

$$(1-e^{-2r})^2 > 4\tau_{in}(1-\tau_{in}). \quad (34)$$

When Inequality (34) is fulfilled, the region for the positive $\tau_D(T)$ is given by

$$\frac{(1-e^{-2r}) - \sqrt{(1-e^{-2r})^2 + 4\tau_{in}(\tau_{in}-1)}}{2\tau_{in}}$$
$$< T < \frac{(1-e^{-2r}) + \sqrt{(1-e^{-2r})^2 + 4\tau_{in}(\tau_{in}-1)}}{2\tau_{in}}. \quad (35)$$

Since T given in Inequality (35) must fulfill $0 \leq T \leq 1$ simultaneously, τ_{in} in Inequalities (34) and (35) have the condition of $1/2 \leq \tau_{in} \leq 1$. Dependence on T for nonclassical depths of $\tau_{out}^{tel}(r, T)$ and $\tau_{out}^{dir}(T^2)$ and the bound (34) are illustrated in Fig.4. For the input state with the maximal nonclassicality $\tau_{in} \to 1$, Inequality (35) is simplified to

$$0 < T < (1-e^{-2r}), \quad (36)$$

and it gives the positive $\tau_D(T)$ for all T in the strong squeezing limit ($r \to \infty$).

These results show that the better choice to transmit the nonclassicality of the quantum states still depends on the loss of the quantum channel and the nonclassical depth of the transferred quantum state itself.

7 Conclusion

In conclusion, we have studied the transmission performance of nonclassical states by the noisy teleportation channel with respect to the nonclassical depth. The noise is assumed that the coupling between the system and the vacuum field. The results are compared to those of the direct transmission channel and we find that the teleportation is better than the direct transmission in a certain region. Physically decoherence mechanisms are different between these two channels. The decoherence in teleporation is effectively described by a thermalization process with the averaged photon number $\bar{n}_{\lambda T}$ although the loss model is assumed to be the interaction with a vacuum environment.

We finally apply our results to the realistic situations in experiments. In the experiment of [6], the fidelity of $F = 0.58 \pm 0.02$ was achieved for the coherent state teleportation with the amplitude efficiency of 0.9 for each two-mode squeezed-vacuum delivery. Due to the technical limit, the squeezing used for teleportation was limited to 3 dB although the maximum squeezing of 6 dB was already observed in the same experimental setup[13]. Substituting $T = 0.81$ and the effective squeezing of 3 dB ($r = 0.34$) into Eq. (19), we obtain $F = 0.62$ for the coherent state teleportation. Although the fidelity obtained is slightly overestimated, our theoretical result shows a reasonable agreement.

Now we consider a transmission of the 6 dB squeezed state (which corresponds to $r = 0.69$ and $\tau_c^{in} = 0.38$) by the teleportation channel with the 6 dB two-mode squeezed-vacuum. From Eq. (25), we find that the transmittance of $T > 0.83$ is at least necessary for obtaining the nonzero nonclassical depth τ_c^{out} after the teleportation. It is also shown in Fig.4 (B) that such teleportation channel will be better than the direct transmission channel with T^2 for the purpose of transferring highly nonclassical states, such as Fock states.

As shown in this report, the performance of the continuous variable teleportation as a quantum communication channel depends not only on the parameters of the channel, but also on what kind of quantum information we want to send. To find the best way for transmitting quantum information through various possible channels is generally a nontrivial problem. It would be an important future problem to study efficient codings against some practical noise models in both teleportation and direct transmission scenarios.

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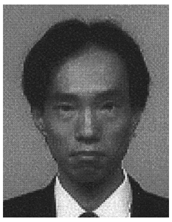
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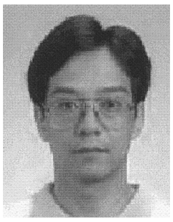
Quantum information theory and quantum optics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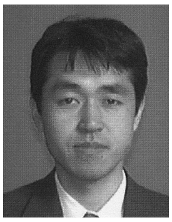
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