# 2-5-5 Determination of Gain for EMC Antennas Using Phase Center

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This chapter describes numerical analyses of the distance-dependent gain variation that exists in gain measurements based on the Friis transmission formula for typical EMC antennas such as standard gain horn and double-ridged guide horn antennas. The analyses are performed by simulating gain measurements using the method of moments with higher-order basis functions. Simulation and experimental results show the effectiveness of using the location of the phase center to accurately determine the far-field gain at reduced antenna separation distances.

# 1 Foreword

With wider use and progress of systems and services using electro-magnetic waves, maintenance of good electromagnetic wave environments is an important issue. For accurate measurements of such electro-magnetic wave environments, that is, of the strengths of electro-magnetic fields, high precision calibration of antenna gain is required. According to the frequency band, NICT provides a calibration service for antenna factor and gain of dipole antennas, standard horn antennas, wide band antennas for EMC measurements, etc. NICT also performs research and development of highly precise measurement technology for antenna measurements, including reduction of measurement uncertainty.

The three-antenna method based on the Friis transmission formula is often used as a method for measuring the far-field gain of antennas[1]. However, even if the distance between antennas satisfies the far-field conditions, the gain obtained differs depending on the measurement distance for many wide band antennas. For example, in the case of horn antennas, even if the well-known far-field criteria of  $2D^2/\lambda$  (D: Maximum dimension of aperture,  $\lambda$ : wavelength), the gain obtained is approximately 1 dB less than "true" gain. Therefore, the correction coefficient for the gain reduction based on the measurement distance including until the near-field region has been studied[2]-[5]. Especially Chu and Semplak[3] expressed a correction for the gain reduction of a horn antenna as a function of the antenna's dimensions and distance between apertures. For accurate measurements, i.e., to obtain a gain reduction within 0.05 dB, a distance between antennas of approxi-

mately 32  $D^2/\lambda$  is required. Newell et al.[6] proposed an extrapolation technique that enables accurate measurements at shorter distances, from 1/5 to 1/10 the distances required in conventional methods. Since then, many EMC test laboratories adopted a three-antenna method using an extrapolation technique. On the other hand, as other techniques to shorten measurement distances, measurement techniques that consider the phase center are being studied[7]-[14]. The given antenna's reference point (for example, aperture) is usually used as a base point for setting distance. However, the reference point is determined for convenience of use, which differs from the phase center treated as the equivalent point source. It has been shown that gain reduction is caused by difference in distance between the reference point and phase center[14]. Measurement of the phase center requires accurate measurement of the phase pattern, which is not easy[15]. Meanwhile, Muehldorf[16] obtained the theoretical formula of the phase center for a horn antenna. Also, even for a complex structure antenna, numerical analysis by a commercial electromagnetic field solver can be used[17]. The validity of these calculated values of phase centers have been verified in numerical simulation and experimentally[14]. Antenna design is usually performed by CAD, thus antenna structure is accurately represented in an electromagnetic field solver. In this case, phase center can be calculated easily, such as antenna characteristics of gain, directionality and reflection.

This chapter shows the effect of the distance between antennas on the gain measurements based on the Friis transmission formula by numerical simulation using the method of moments, for a typical EMC measurement antenna. Simulation and experimental results show the effectiveness of a gain determination method using the phase center.

## 2 Gain measurement method

The Friis transmission formula is often used to measure gain at far-field[1]. As shown in Fig. 1, the transmitting and receiving antennas face each other separated by only distance *r* in free space. At this time, the multiple of the actual gains of the transmitting and receiving antennas  $G_{w(t)}$ •  $G_{w(t)}$  is expressed by the Friis transmission formula in the equation below.

$$G_{w(t)} \cdot G_{w(r)} = \frac{P_{(r)}}{P_{(t)}} \left(\frac{4\pi r}{\lambda}\right)^2 \tag{1}$$

Here,  $P_{(t)}$  is transmitted power,  $P_{(r)}$  is received power.

Actual gain can be obtained by Equation (1) from measurements of antenna insertion loss  $A_{(tr)}$  (=  $P_{(r)}/P_{(t)}$ ) using a combination of three antennas (Antennas #1 to #3). For example, the gain of Antenna #1 is obtained from the following equation, after solving the system of equations of Equation (1).

$$G_{w(1)} = \frac{4\pi r}{\lambda} \sqrt{\frac{A_{(12)} \cdot A_{(13)}}{A_{(23)}}}$$
(2)

Here, the subscripts are combinations of Antennas #1, #2 and #3. If we assume the two antennas used have the completely same characteristics, then from Equation (1), the actual gain is:

$$G_w = \frac{4\pi r}{\lambda} \sqrt{A} \tag{3}$$

These methods are each called the three-antenna method and two-antenna method[15]. For actual gain, the reflection loss due to impedance mismatch of the antenna input port is considered. If  $\Gamma_{in}$  is reflection coefficient at the input port, then gain *G* is in the following equation.

$$G = \frac{G_w}{1 - \left|\Gamma_{\rm in}\right|^2} \tag{4}$$

In antenna measurements, the transmitting and receiv-



Fig. 1 Friis transmission formula

ing antennas are arranged such that the far-field criteria are satisfied. The minimum far-field criteria  $r \ge 2 D^2 /\lambda_{min}$ (here,  $\lambda_{min}$  is the minimum wavelength) is used widely for aperture antennas[18]. When the size of the aperture of transmitting and receiving antennas cannot be ignored, far-field criteria  $r \ge 2 (D_t + D_r)^2 /\lambda_{min}$  is generally applied (here,  $D_t$  and  $D_r$  are the maximum dimensions of the aperture of the transmitting and receiving antennas)[19][20].

## 3 Simulation of gain measurements

Gain changes due to measurement distance between antennas are assessed by numerical simulation of measurements. In numerical calculation, one can assume the antennas are completely the same, thus we apply the two-antenna method. That is, from Equation (3) and Equation (4), gain  $G^{(r)}$  obtained at distance r is expressed by the following equation using S-parameters ( $S_{21}$  and  $S_{11}$ ) equivalent to insertion loss between antenna ports and reflection loss due to impedance mismatch of antenna ports.

$$G^{(r)} = \frac{4\pi r}{\lambda} \frac{|S_{21}|}{1 - |S_{11}|^2}$$
(5)

Here, for distance between antennas, the distance between given reference points in the antennas is used (for example, apertures).

In Sections **3.1** and **3.2**, we assess how distance between antennas affects gain measurement of a standard gain horn antenna and double-ridged guide horn (DRGH), by performing a numerical simulation using WIPL-D[21], fullwave electromagnetic field solver based on the method of moments with higher-order basis function[14][26].



Fig. 2 Pyramidal horn antenna

#### 3.1 Standard gain horn antenna

A standard gain horn antenna is often used as a reference antenna. Figure 2 shows the structure of a typical pyramidal standard horn antenna. Numerical simulation of gain measurements of a horn antenna was performed by the method of moments using a basis function by quartic polynomial[26].

Figure 3 shows a calculation model for a C-band (5.85–8.2 GHz) horn using calculations. Two horn antennas with the same dimensions were placed facing each other, with only distance *r* between their apertures. The antenna model assumes a perfect conductor with no thickness, and it is excited in the fundamental mode of the rectangular waveguide (TE<sub>10</sub>). The simulation model is comprised of quadrilateral patches with a maximum length of one minimum wavelength ( $\lambda_{min}$ ). Figure 4 shows gain



Fig. 3 Simulation model of gain measurement for C-band pyramidal horn by the method of moments (a = 288 mm, b = 213 mm,  $l_{E} = 481.9$  mm, and  $l_{H} = 515.4$  mm)[26]



Fig. 4 Distance dependency of gain for C-band pyramidal horn simulated by the method of moments[26]

results determined by numerical simulation using the method of moments, at each distance between apertures, from 32  $D^2 / \lambda_{\min}$  (72.55 m) to  $D^2 / \lambda_{\min}$  (2.27 m). We see that in gain measurements based on the two-antenna method, that is, the Friis transmission formula, gain obtained depends on the measurement distance. To accurately determine far-field gain, sufficient distance is required (for example, 32  $D^2 / \lambda_{\min}$ ). Gain change *dG* is defined as this gain  $G^{(r)}$  divided by far-field gain  $G^{FAR}$ .

$$dG = \frac{G^{(r)}}{G^{EAR}} \tag{6}$$

Chu and Semplak[3] calculated a gain reduction correction value (the inverse of dG) which is a function of the antenna's dimensions and distance between apertures.

Figure 5 shows the gain reduction for far-field gain due to distance between antennas obtained from simulation by the method of moments. The same figure shows a comparison with the correction value of gain reduction by Chu. For example, in the  $D^2 / \lambda_{min}$  distance between them, ripple shape fluctuations occur due to the effects of reflection waves between antennas. Chu's correction values closely match the simulation results, but we see that such effects of reflection waves are ignored. Also, these results show that even if the well-known far-field criteria 2  $D^2 / \lambda_{min}$  is satisfied, the gain decreases by approximately 0.8 dB, and to make the gain reduction be 0.05 dB, distance of 32  $D^2 / \lambda_{min}$  or more is required.

#### 3.2 Double-ridged guide horn antenna

DRGH is often used as a wide band antenna in EMC measurements. Such as described in the previous chapter, we use the method of moments to assess the distance



Fig. 5 Gain reduction due to distance between apertures of C-band pyramidal horn simulated by the method of moments[14]





(b)

characteristics of gain of DRGH[14]. Figure 6 (a) shows the calculation model for DRGH. The antenna model assumes a perfect conductor, and it is excited by a coaxial feed. The designed frequency bandwidth is 1 GHz to 12 GHz. Figure 6 (b) shows the simulation model for gain measurements. That is, antennas with the same dimensions are placed facing each other, separated by only distance rbetween apertures. The simulation model shown in Figure 6 is comprised of quadrilateral patches with a maximum length of one minimum wavelength. The basis function used is a quartic polynomial; the unknown number overall at this time is 17,595, using the symmetric structure of the antenna. The gain is determined from Equation (5) by calculating the S-parameters at both antenna ports for several separation distances.

Figure 7 shows simulation results of gain measurement by the method of moments. Gain determined by the Friis transmission formula changes according to the distance



Fig. 7 Distance dependency of gain for DRGH simulated by the method of moments[14]



Fig. 8 FIM model of C-band pyramidal horn antenna (s = 408 mm, t = 333 mm, and u = 628 mm)

between antennas. To obtain the far-field gain, sufficient distance is required, for example 15 m. Also, the direction of maximum gain in the 8 GHz to 12 GHz range sometimes differs from the bore sight direction in front of the antenna; this directionality characteristic is often observed in ridged guide horns with similar structure[22][23].

#### 3.3 Phase center

The antenna phase center is defined as the center of curvature of the equiphase surface of the radiation waves at the far-field. Measurement of the phase center requires accurate phase measurement and equipment for antenna scanning; it is generally difficult[15]. On the other hand, in the numerical approach, the phase center can be computed from the equiphase pattern by adjusting the origin of the near- to far-field transformation. The electromag-



Fig. 9 H-plane phase pattern of C-band pyramidal horn antenna obtained by FIM[14]



Fig. 10 Phase center of C-band pyramidal horn antenna obtained by FIM[14]

netic field solver (CST MS-studio)[17] based on the finite integration method (FIM) has such a post process that can determine the phase center. The phase center position of the C-band horn shown in Fig. 2 was calculated by an FIM solver. Figure 8 shows an FIM model of a horn antenna. We model the analysis area into 15,763,986 (=  $183 \times 147 \times 586$ ) nonuniform cells with maximum cell size  $\lambda_{min}$ / 20, and used a PML[24] with 8 layers as the absorbing boundary. We assume the antenna material is a perfect conductor, and excited it in the fundamental mode TE<sub>10</sub> of the rectangular waveguide.

Figure 9 shows an example calculation (6 GHz) of a phase pattern positioned in the phase center and its anteroposterior locations. We see there is almost no change in phase when the antenna's scanning range is approximately half of the beam width, that is, in the range  $\pm 2$  degrees (=  $\theta$ ). Therefore, we calculated the phase center in  $\pm 2$  degrees scanning range, for each frequency from



Fig. 11 Relation between gain reduction and ratio of distance between apertures to distance between phase centers for pyramidal horns[14]

5.8 GHz to 8.2 GHz. The phase center positions  $d_H$  and  $d_E$ of H- and E-planes are obtained as distances from the aperture on the bore sight axis. Figure 10 shows the calculation results by FIM of these phase centers. We see that the phase centers are not on the aperture; they move towards the waveguide's port along with increase in frequency. Figure 10 also shows and compares vs. Muehldorf's theoretical values[16]. FIM's calculation results almost match the theoretical values. The slight difference may be due to the effect of internal reflections from the discontinuities in the antenna such as from the edges of the aperture. The average phase center of both planes,  $d_{pc}$  (= ( $d_H$  +  $d_E$ )/2), is considered because it coincides with the location of the amplitude center[25]. In other words, it can be treated as the equivalent point source when the antenna is observed in far-field. Hereinafter in this chapter, the average phase center is used as the "phase center."

We verify the relationship between the position reference and gain changes that occur due to distance between antennas in gain measurements. For commercial horns from C- to W-bands, Fig. 11 compares the ratio ( $\Delta$ ) of the distance (r) between apertures divided by the distance between phase centers ( $r + 2 d_{pc}$ ) vs. Chu's calculated values of gain reduction. Here, the phase center of each horn is the value calculated by FIM. This result shows that if the measurement distance approximately is 4 D<sup>2</sup> / $\lambda_{min}$  separated, then the distance ratio  $\Delta$  is very near the gain reduction. That is, from Equation (6),

$$dG = \frac{G_{(r)}}{G_{FAR}} \cong \frac{r}{r + 2 \cdot d_{pc}}$$
(7)

Figure 12 shows gain changes due to measurement



Distance between apertures, r (m)

Fig. 12 Distance characteristics of gain for C-band pyramidal horn simulated by the method of moments[26]



Fig. 13 Phase center of C-band pyramidal horn[14]

distance of C-band horn. Gain is determined by measurement simulation by the method of moments of the twoantenna method. If we assume that such gain changes in far-field gain are equal to the ratio of distances between apertures and between phase centers, then we can estimate the phase center from the distance dependency of gain. That is, in the range of distances that satisfies far-field, we obtain the phase center  $d_{PC}$  from curve fitting by leastsquares method using the following equation[26][27].

$$G(r), d\mathbf{B} = 10 \cdot \log \frac{r}{r+2 \cdot a} + b \tag{8}$$

Figure 12 shows a fitting curve using this function. For the range from 30 m to 80 m which satisfies far-field, we fitted this by the Levenberg-Marquardt algorithm using 126 data in 0.4 m steps. a and b of the fitting curve obtained correspond to the phase center and far-field gain. For example, from the 8.2 GHz result shown in the figure, we obtain 0.426 m (a) and 22.88 dBi (b). Figure 13 shows



**Fig. 14** Distance characteristics of gain for DRGH simulated by the method of moments[14]



Fig. 15 Phase center of DRGH[14]

phase centers estimated in this way by fitting from distance characteristics of gain for each frequency. This result is in agreement with values calculated by Muehldorf's[14] theoretical formula and FIM. That is, the assumptions are valid for Equation (7) regarding gain changes.

Next, we performed a similar assessment for the DRGH shown in Figure 6 (b)[14]. Figure 14 shows changes in gain due to distance between antennas that occur in gain measurements. We changed distance in 0.2 m steps from 0 m to 15 m, and determined gain by simulation of the twoantenna method by the method of moments shown in Figure 6 (b). Figure 15 shows results from phase centers estimated for each frequency, from gain changes in fitting using Equation (8). The phase centers are determined by fitting in the range from 3 m to 15 m, which satisfies farfield. These results closely match results of calculating FIM estimated from phase patterns. The phase center of DRGH changes intricately compared to a standard horn, and located outside the antenna in a frequency band 9 GHz or



Fig. 16 Simulation results of (a) gain of C-band pyramidal horn and (b) gain of DRGH determined using phase center[14]

higher. In that frequency band, this is due to the complexity of directivity, such as when the antenna main lobe does not face towards the antenna front[22][23].

### 3.4 Application of phase center

The gain variation due to the distance between antennas that occurs in gain measurements is caused by the difference between the given reference point (for example, the aperture) and the phase center. That is, application of the phase center as the antenna location reference point is appropriate. Figure 16 shows simulation results of gain of a standard horn and DRGH, using distance between phase centers as distance between antennas. We see that gain exhibit good agreement with far-field gain, regardless of the separation distance. These results show that by considering the phase center in gain measurements, gain changes do not occur due to the distance between antennas as shown in Fig. 4 and 7. That is, these indicate that shorter



Fig. 17 Measured gain of (a) C-band pyramidal horn and (b) DRGH by the three-antenna method[14][28]

measurement distance is possible. For example, for a horn antenna, one can shorten down to approximately 1/8 (4  $D^2$  / $\lambda$ ) compared to the distance required in accurate measurements by conventional methods (32  $D^2$  / $\lambda$ ). Even if an extrapolation technique[6] is used, similar shorter distances are possible, and consideration of distance criteria is not required. However, when measuring radiated emissions at relatively near distances (for example, 1 m), it is useful to have the phase center in settings criteria of measurement distance.

# 4 Experimental verification

We experimentally verified the effectiveness of the phase center technique, using two different types of commercial antennas. That is, in an anechoic chamber, V-band (50–75 GHz) pyramidal horn antennas were arranged facing each other with 1.32 m (4  $D^2$  / $\lambda_{min}$ ) between aper-

tures, and connected to a network analyzer. By combining three antennas, we measured insertion loss ( $S_{21}$  parameter) and impedance mismatch loss ( $S_{11}$  parameter), and determined gain by the three-antenna method using Equations (2) and (4). For a DRGH that has the 1 GHz to 18 GHz band, we similarly measured gain at 3 m distance between apertures[28]. We also calculated the phase centers of these antennas, from phase patterns by using an FIM solver.

Figure 17 shows results of measuring gains of a pyramidal horn and DRGH, obtained by the three-antenna method. Figure 17 also shows theoretical values of gain obtained by FIM. Horn antenna gain using distance between apertures as distance between antennas (4  $D^2 / \lambda_{min}$ ) is approximately 0.4 dB less than theoretical values as shown in Fig. 17 (a). However, results while applying distance between phase centers in the distance between the antennas are in good agreement with theoretical values. Also, even in the case of DRGH, results from using phase center as the antenna's location reference closely match theoretical values, even at a nearby 3 m measurement distance. If aperture is the reference, then one cannot ignore gain changes based on differences in distance vs. the phase center. These results confirmed the effectiveness of applying phase center to accurately determine gain in measurement distances that are shorter than conventional methods.

# 5 Conclusion

We used simulation of measurements by the moments of method to assess gain fluctuations caused by distance between antennas of gain measurements, for standard gain horn and double-ridged guide antennas which are typically used in measurements. Numerical simulation results show that (1) Gain fluctuations occur in conventional methods, even if far-field criteria are satisfied; (2) These gain changes are equivalent to the ratio of distance between reference points vs. distance between phase centers; (3) Gain determined using distance between phase centers closely matches far-field gain, regardless of the measurement distance. That is, by using distance between phase centers, one can shorten the measurement distance required for accurate measurements in conventional methods (for example, in the case of a horn antenna, approximately 1/8 the distance). Moreover, experiments using commercial antennas confirmed the effectiveness of the phase center technique. We plan to study application of phase centers in EMC measurements at relatively close distances.

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